Lösung 11.3

geg.: $1 \rightarrow 2$ Reaktion in der Brennkammer mit anschließender Strömung durch eine Lavaldüse

$\vartheta_{1,O_2} = \vartheta_{1,H_2} = 127\,°\mathrm{C}$	$\vartheta_2 = 1727\,°\mathrm{C}$	$\lambda = 1$
$p_{1,O_2} = p_{1,H_2} = 8\,\mathrm{bar}$	$p_2 = 1\,\mathrm{bar}$	$\dot{m}_{H_2} = 40\,\frac{\mathrm{kg}}{\mathrm{h}}$
$\dot{Q}_{12} = -1844\,\frac{\mathrm{MJ}}{\mathrm{h}}$	$T_u = 300\,\mathrm{K}$	$c_2 > c_s$
$Oxidator\; O_2$	$\Delta e_{pot} = 0$	$e_{a_1} = 0$

a) $\dot{n}_{1,H_2} \qquad \dot{n}_{1,O_2}$

$$\lambda = 1 \Rightarrow H_2 + \tfrac{1}{2} O_2 \rightarrow H_2O$$

$$\dot{n}_{1,H_2} = \frac{\dot{m}_{H_2}}{M_{H_2}} = \frac{40\,\frac{\mathrm{kg}}{\mathrm{h}}}{2\,\frac{\mathrm{kg}}{\mathrm{kmol}}} = 20\,\frac{\mathrm{kmol}}{\mathrm{h}}$$

$$\dot{n}_{1,O_2} = \tfrac{1}{2} \cdot \dot{n}_{1,H_2} = 10\,\frac{\mathrm{kmol}}{\mathrm{h}} \qquad \dot{n}_{2,H_2O} = \dot{n}_{1,H_2} = 20\,\frac{\mathrm{kmol}}{\mathrm{h}}$$

b) c_2

$$\dot{Q}_{12} = \dot{H}_2 - \dot{H}_1 + \dot{m}_2 \cdot \frac{c_2^{\,2}}{2}$$

$$\dot{H}_2 = \dot{n}_{2,H_2O} \cdot h_{m_{2,H_2O}}$$

$$\dot{H}_1 = \dot{n}_{1,H_2} \cdot h_{m_{1,H_2}} + \dot{n}_{1,O_2} \cdot h_{m_{1,O_2}}$$

$$\dot{m}_2 = \dot{n}_{2,H_2O} \cdot M_{H_2O}$$

$$c_2 = \sqrt{2 \cdot \frac{\dot{Q}_{12} - \left(\dot{H}_2 - \dot{H}_1\right)}{\dot{n}_{2,H_2O} \cdot M_{H_2O}}} = \sqrt{2 \cdot \left[\frac{\dot{Q}_{12}}{\dot{n}_{2,H_2O} \cdot M_{H_2O}} - \frac{1}{M_{H_2O}} \cdot \left(h_{m_{2,H_2O}} - h_{m_{1,H_2}} - \frac{1}{2} \cdot h_{m_{1,O_2}}\right)\right]}$$

$$h_{m_{2,H_2O}} = -155{,}92\,\frac{\mathrm{MJ}}{\mathrm{kmol}}$$

$$h_{m_{1,H_2}} = 11{,}43\,\frac{\mathrm{MJ}}{\mathrm{kmol}}$$

$$h_{m_{1,O_2}} = 11{,}71\,\frac{\mathrm{MJ}}{\mathrm{kmol}} \qquad \Rightarrow c_2 = 3000\,\frac{\mathrm{m}}{\mathrm{s}}$$

c) $e_{m_{V12}}$

$$e_{m_V} = \frac{\dot{E}_V}{\dot{n}_{1,H_2}} = T_u \cdot \frac{\dot{S}_{irr}}{\dot{n}_{1,H_2}}$$

$$\dot{S}_2 - \dot{S}_1 = \dot{S}_{irr_{12}} + \dot{S}_{aust_{12}} \Rightarrow \dot{S}_{irr_{12}} = \dot{S}_2 - \dot{S}_1 - \dot{S}_{aust_{12}}$$

$$\dot{S}_2 - \dot{S}_1 = \dot{n}_{2,H_2O} \cdot s_{m_{2,H_2O}} - \dot{n}_{1,H_2} \cdot s_{m_{1,H_2}} - \dot{n}_{1,O_2} \cdot s_{m_{1,O_2}} \qquad p_2 = p_0$$

mit $s_{m_i}(p,T) = \underbrace{s_{m_i}(p_0,T)}_{=s_{m_i}^0} - R_m \cdot \ln \frac{p_i}{p_0}$

$$\dot{S}_2 - \dot{S}_1 = \dot{n}_{2,H_2} \cdot \left[s^0_{m_{2,H_2O}} - s^0_{m_{1,H_2}} - \frac{1}{2} \cdot s^0_{m_{1,O_2}} + R_m \cdot \left(\ln \frac{p_1}{p_0} + \frac{1}{2} \cdot \ln \frac{p_1}{p_0} \right) \right] \text{ und } \dot{S}_{aust_{12}} = \frac{\dot{Q}_{12}}{T_u}$$

$$e_{m_V} = -\frac{\dot{Q}_{12}}{\dot{n}_{1,H_2}} + T_u \cdot \left(s_{m_{2,H_2O}} - s_{m_{1,H_2}} - \frac{1}{2} \cdot s_{m_{1,O_2}} + \frac{3}{2} \cdot R \cdot \ln \frac{p_1}{p_0} \right)$$

$$= \left[\tfrac{1844}{20} + 300 \cdot \left(0,26502 - 0,13921 - \tfrac{1}{2} \cdot 0,21387 + \tfrac{3}{2} \cdot 8,3148 \cdot 10^{-3} \cdot \ln \tfrac{8}{1} \right) \right] \tfrac{\text{MJ}}{\text{kmol}}$$

$$= 105,6 \, \tfrac{\text{MJ}}{\text{kmol}}$$

Lösung 11.7

geg.:

$p_{3,I} = 0,2\,\text{bar}$ $\quad T_{3,I} = T_u = 300\,\text{K}$ $\quad p_u = 1\,\text{bar}$

$\Delta\tau_{I,II} = 10\,\text{s}$ $\quad c_2 = 0,8 \cdot c_{s_2}$ $\quad A_2 = 0,5\,\text{m}^2$

$p_2 = const.$ $\quad n = 1,4 = \kappa$ $\quad |\text{Idealgas}$

$R = 0,287\,\frac{\text{kJ}}{\text{kg×K}}$ $\quad c_p = 1,004\,\frac{\text{kJ}}{\text{kg×K}}$

a) T_2 $\quad p_2$ $\quad c_2$ $\quad m$

$$c_2 = 0,8 \cdot c_{2_2} = 0,8 \cdot \sqrt{\kappa \cdot R \cdot T}$$

1.HS: $$0 = h_2 - h_1 + \frac{c_2^{\,2}}{2}$$

$$\Rightarrow c_2^{\,2} = -2 \cdot c_p \cdot \left(T_2 - T_1\right) = 0,8^2 \cdot \kappa \cdot R \cdot T_2$$

$$\Rightarrow T_2 = \frac{2 \cdot c_p \cdot T_1}{0,64 \cdot \kappa \cdot R + 2 \cdot c_p} = 266\,\text{K}$$

$$p_2 = p_1 \cdot \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = 0,656\,\text{bar} \qquad \text{c}_2 = 0,8 \cdot \sqrt{\kappa \cdot R \cdot T_2} = 261,5\,\tfrac{\text{m}}{\text{s}}$$

$$m = c_2 \cdot A_2 \cdot \rho \cdot \Delta t_{I,II} = c_2 \cdot A_2 \cdot \frac{p_2}{R \cdot T_2} \cdot \Delta t_{I,II} = 1123,5\,\text{kg}$$

c) $T_{3,II}$ $\quad p_{3,II}$

$$V_3 = 7000\,\text{m}^3$$

1.HS um gesamte Anlage:

$$dU = \sum_i h_{t_i}\, dm_i = h_1 dm_1$$

$$\rightarrow \int_{3,I}^{3,II} dU = h_1 \cdot \int_{3,I}^{3,II} dm$$

$$\rightarrow \Delta U_{3,II \rightarrow I} = h_1 \cdot \underbrace{\left(m_{3,II} - m_{3,I}\right)}_{=m}$$

$$\rightarrow m_{3,II} \cdot c_v \cdot T_{3,II} - m_{3,I} \cdot c_v \cdot T_{3,I} = c_p \cdot T_1 \cdot m$$

$$\rightarrow T_{3,II} = \frac{m_{3,I} \cdot c_v \cdot T_{3,I} + m \cdot c_p \cdot T_1}{m_{3,II} \cdot c_v} = \frac{m_{3,I} \cdot T_{3,I} + m \cdot \kappa \cdot T_1}{m_{3,II}}$$

$$m_{3,I} = \frac{V_3 \cdot p_{3,I}}{R \cdot T_{3,I}} = 1626\,\text{kg} \rightarrow \text{T}_{3,II} = 349\,\text{K}$$

$$p_{3,II} = \frac{T_{3,II} \cdot R \cdot m_{3,II}}{V_3} = 0,393\,\text{bar}$$