Universität Rostock

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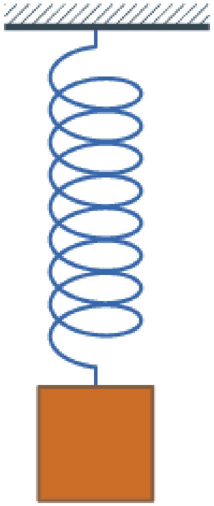
Vibration and Natural Frequencies in Turbomachinery System

Outline

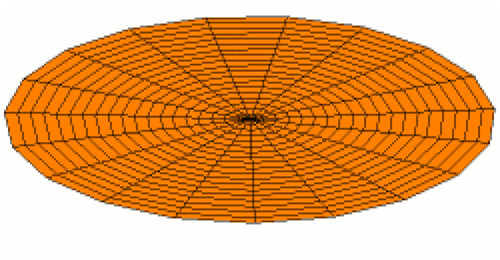
- Vibration
- Natural frequency
- Resonance
- Tacoma bridge
- Rotordynamics
- Numerical Techniques
- Vibration measurement

Vibration:

- Vibration is the result of energy being transferred back and forth between kinetic and potential energies. Vibration refers to mechanical oscillations about an *equilibrium point*.



Vibration of spring damper system



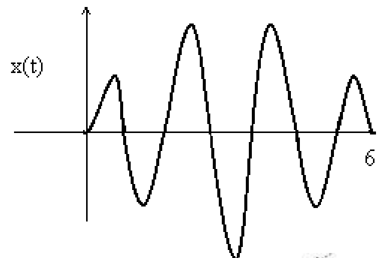
Vibration of circular disk

Why do we care about vibration?

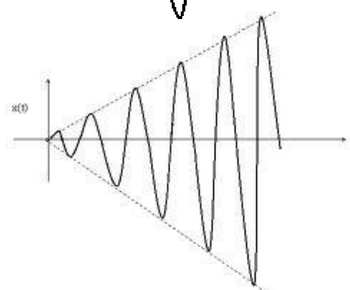
- Vibration is wasted energy
- A major cause of premature component failure
- Cause of noise contribute to discomfort

Vibration:

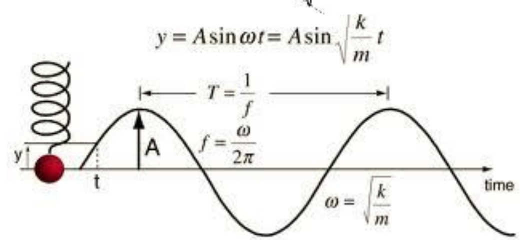
- Vibration
  - Periodic
  - Resonant
  - Harmonic
  - Random



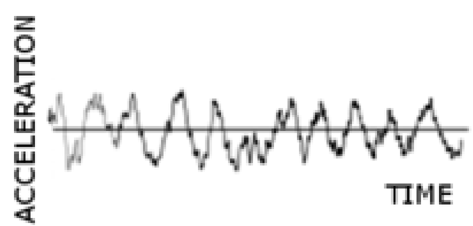
Example:
Rotation of disk having mass imbalance



Example:
Tacoma bridge

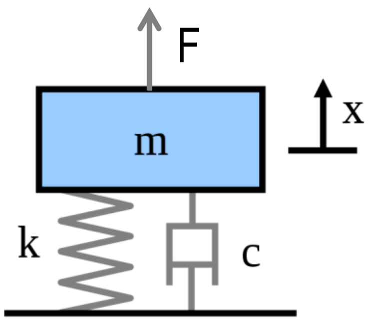


Example:
Pendulum or spring mass system



Example:
the movement of a tire on a gravel road.

General equation of Vibration:



$$ma = -kx - c\dot{x} + \mathrm{F}$$

$k:$ *Spring stiffness*
$c:$ *Damping coefficient*
$F:$ *Force*
$m:$ *Mass*

- Vibration
  - Free Vibration
    - Damped: $ma = -kx - c\dot{x} + \cancel{\mathrm{F}}$ $\quad ma + c\dot{x} + kx = 0$
    - Undamped: $ma = -kx - \cancel{c\dot{x}} + \cancel{\mathrm{F}}$ $\quad ma + kx = 0$
  - Forced Vibration
    - Damped: $ma = -kx - c\dot{x} + \mathrm{F}$ $\quad ma + c\dot{x} + kx = \mathrm{F}$
    - Undamped: $ma = -kx - \cancel{c\dot{x}} + \mathrm{F}$ $\quad ma + kx = \mathrm{F}$

Damped Vibration:

$$ma + c\dot{x} + kx = 0$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

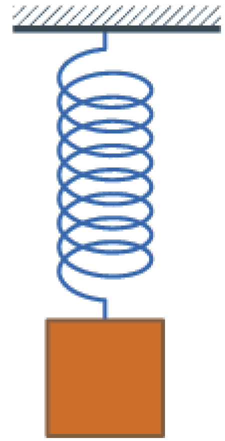
$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0{}^2x = 0$$

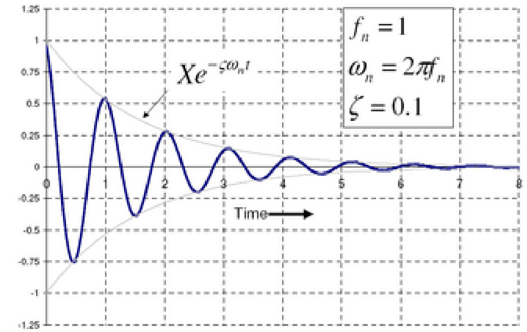
$$\zeta = \frac{c}{2\sqrt{mk}}$$

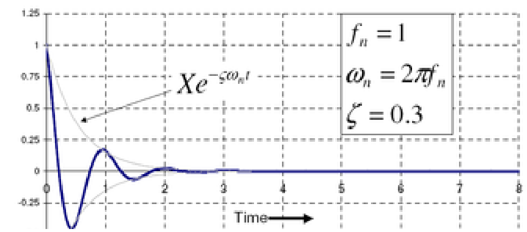
$$\omega_0 = \sqrt{\frac{k}{m}}$$

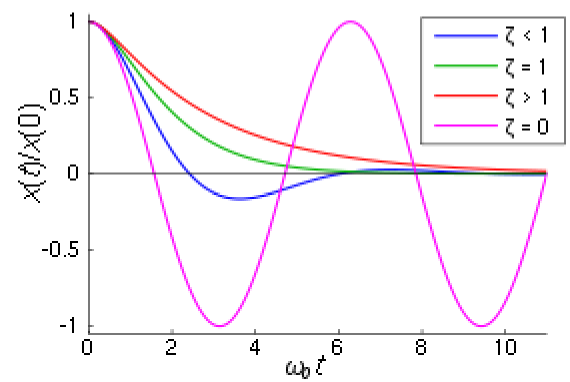
$\zeta = 1 : Critical\ damping$
$\zeta > 1 : Over\ damped$
$0 \leq \zeta < 1 : Under\ damped$

ω_0: Natural frequency







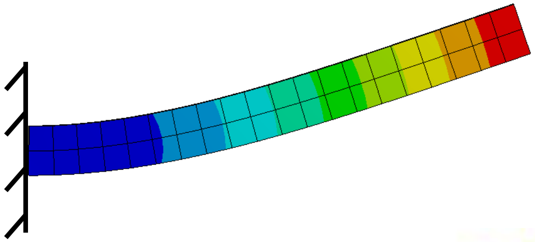


Natural modes and frequency:

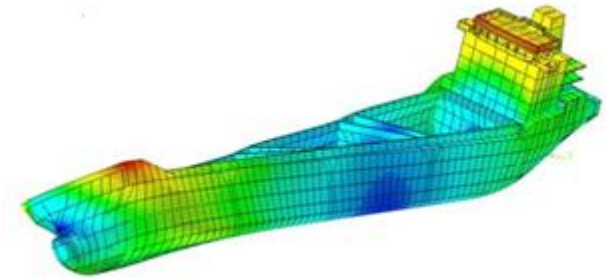
- The natural frequency is the frequency at which a system oscillates when it is disturbed.
- Natural frequency depends of many factors, such as stiffness, mass, length and damping of material etc.
- The natural frequency is important for many reasons:
  - All things in the universe have a natural frequency, and many things have more than one.
  - If you know an object's natural frequency, you know how it will vibrate.
  - If you know how an object vibrates, you know what kinds of waves it will create.
  - If you want to make specific kinds of waves, you need to create objects with natural frequencies that match the waves you want.
  - Predict failure of mechanical structure because of vibration and resonance

Type of modes:

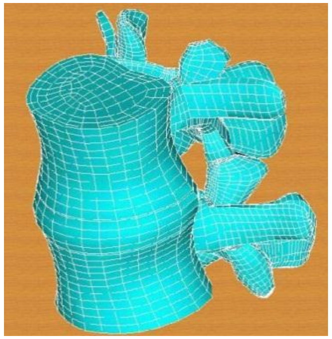
- Lateral bending modes



- Torsional modes



- Axial modes

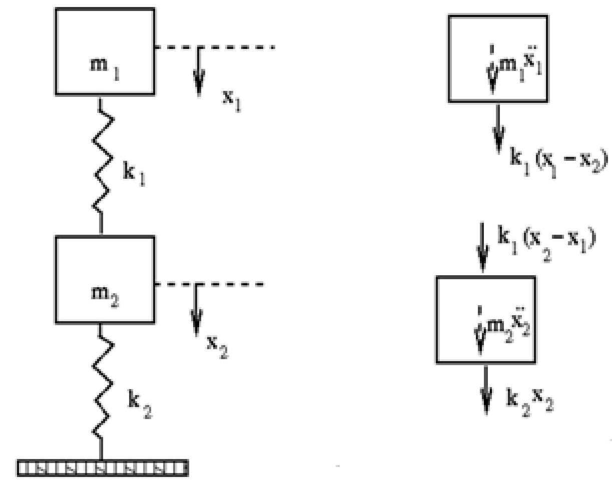


Calculation of frequency:

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) = 0$$

$$m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2x_2 = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$



Let

$x_1 = a_1 \sin wt$ $\qquad \ddot{x}_1 = -a_1 w^2 \sin wt$

$x_2 = a_2 \sin wt$ $\qquad \ddot{x}_2 = -a_2 w^2 \sin wt$

$$\begin{bmatrix} k_1 - m_1w^2 & -k_1 \\ -k_1 & k_1 + k_2 - m_2w^2 \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 0$$

$$\begin{vmatrix} k_1 - m_1w^2 & -k_1 \\ -k_1 & k_1 + k_2 - m_2w^2 \end{vmatrix} = 0$$

Solving determinant we will get frequency of vibration.
By getting ratio of a_1 a_2 we can predict mode shape of vibration

Find frequency and modes.

Assume:
$k_1 = 10$; $m_1 = 5$
$k_2 = 20$; $m_2 = 10$

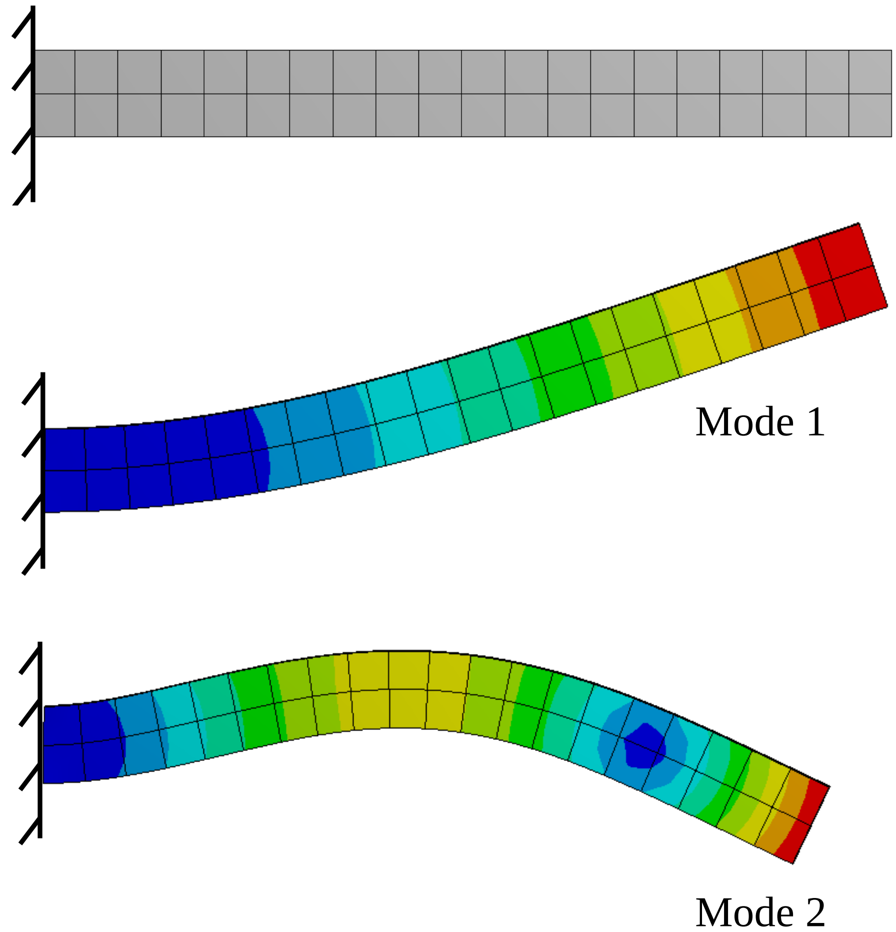
Modal analysis using FEM:

$$[M]\,[\ddot{U}] + [C]\,[\dot{U}] + [K]\,[U] = [F]$$

$$[M]\,[\ddot{U}] + [K]\,[U] = [0]$$

$$[M] = \begin{bmatrix} m_{11} & \cdots & m_{1n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nn} \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{n1} & \cdots & k_{nn} \end{bmatrix}$$

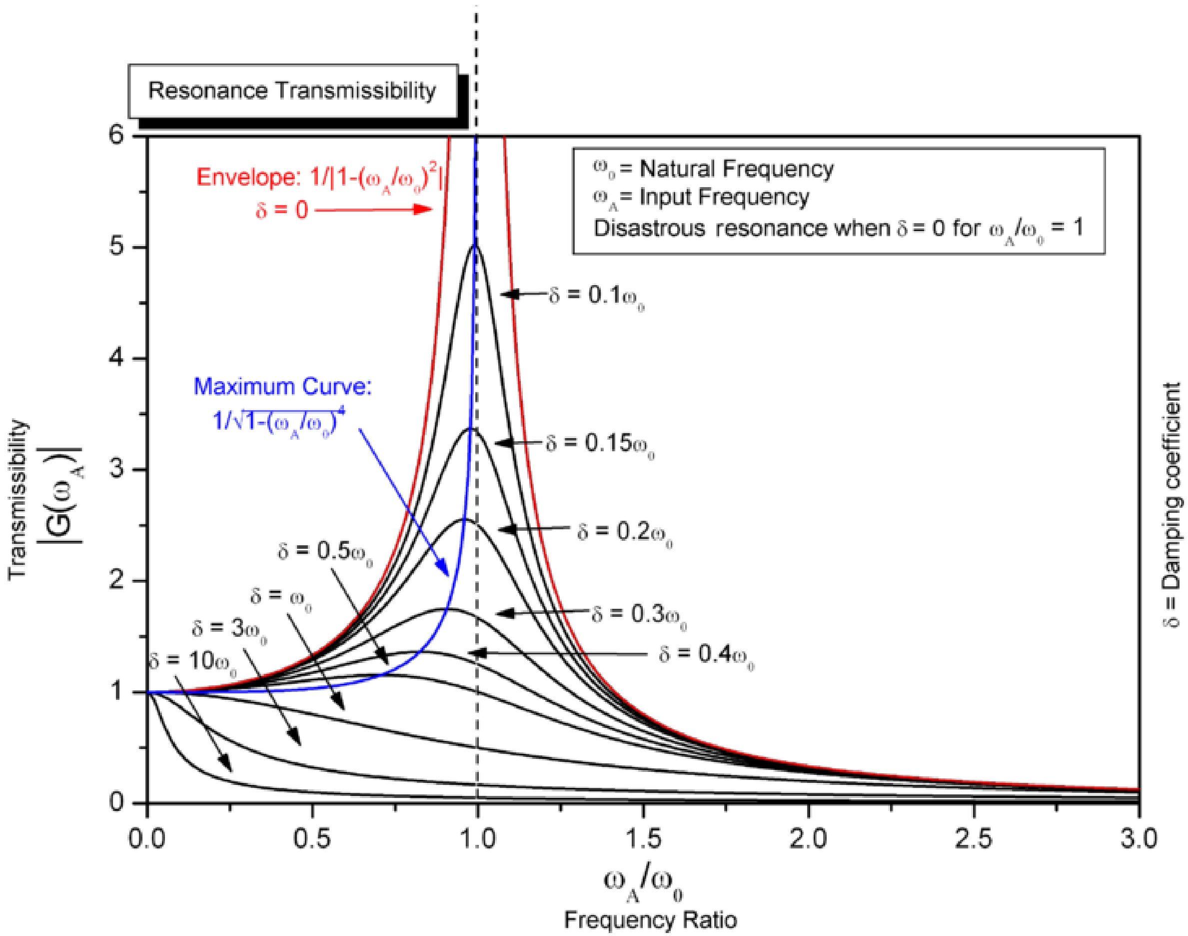


1. How many natural frequencies are possible of given system?
2. Which natural frequency is important?

Resonance:

- **Resonance** is the tendency of a system to oscillate with larger amplitude at some frequencies than at others.
- **Mechanical resonance** is the tendency of a mechanical system to absorb more energy when the frequency of its oscillations matches the system's natural frequency of vibration than it does at other frequencies.
- It may cause violent swaying motions and even catastrophic failure in improperly constructed structures including bridges, buildings, trains, and airplanes.
- Engineers when designing objects must ensure that the mechanical resonant frequencies of the component parts do not match driving vibrational frequencies of the motors or other oscillating parts a phenomenon known as resonance disaster.

Resonance:


Resonance Transmissibility
Envelope: 1/|1-(ωA/ω0)²|
δ = 0
ω0 = Natural Frequency
ωA = Input Frequency
Disastrous resonance when δ = 0 for ωA/ω0 = 1
δ = 0.1ω0
Maximum Curve:
1/√1-(ωA/ω0)⁴
δ = 0.15ω0
δ = 0.2ω0
δ = 0.5ω0
δ = ω0
δ = 3ω0
δ = 10ω0
δ = 0.3ω0
δ = 0.4ω0
Transmissibility
|G(ωA)|
ωA/ω0
Frequency Ratio
δ = Damping coefficient

Tacoma Bridge: Example

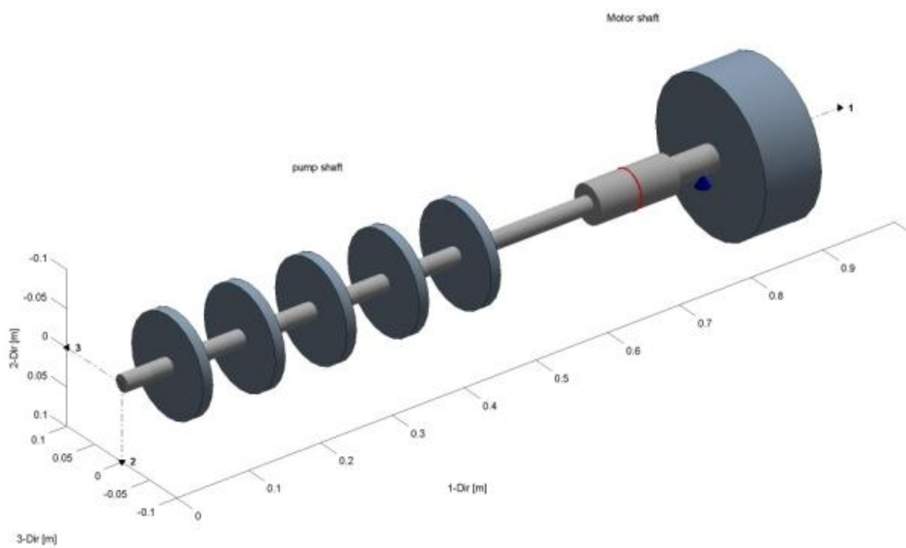
Start of construction: Nov,23,1938

Opened for Traffic: July 1, 1940

Collapse of bridge: Nov,7,1940

Vibration in Turbomachinery system:

- Mechanically Induced
  - Bent Shaft
  - Unbalanced Rotor
  - Bad Bearings
  - Mis-Alignment
  - Looseness



Vibration in Turbomachinery system:

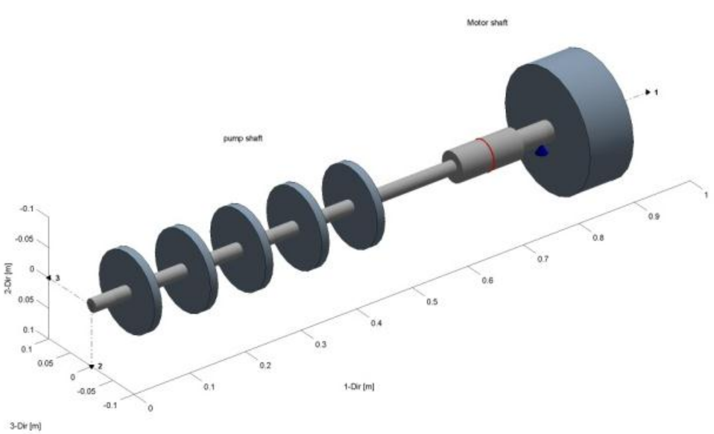
- Operation Induced
  - Cavitation
  - Flow
  - Speed
  - Insufficient Immersion of suction pipe

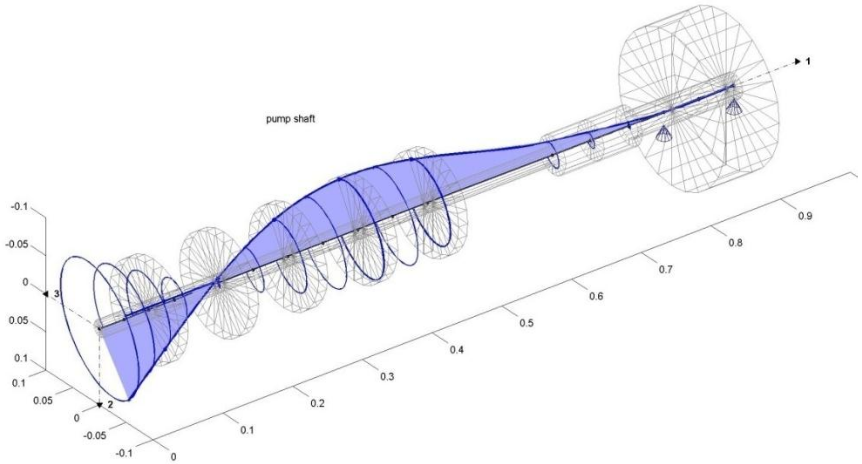
- System Induced
  - Piping
  - Clogged impeller or Suction Line
  - Baseplate

Rotordynamics:

- **Rotordynamics** is a specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures.

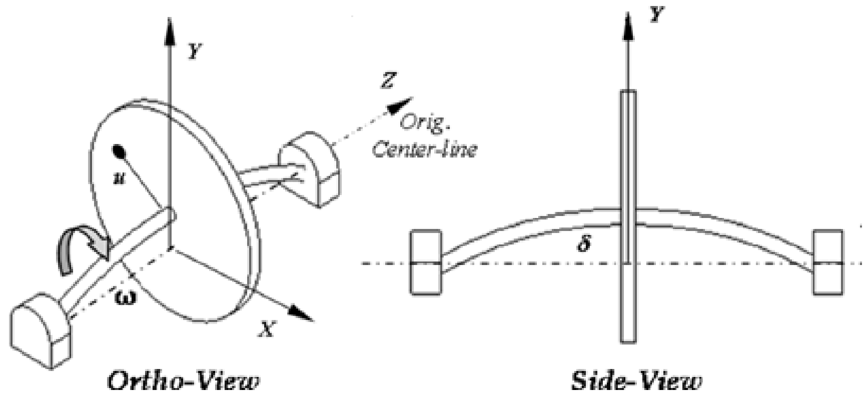
- It is commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage.





Rotordynamics:

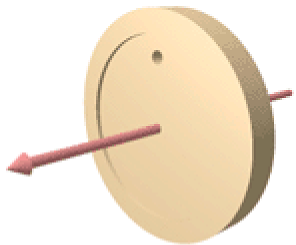
- At its most basic level rotordynamics is concerned with one or more mechanical structures (rotors) supported by bearings and influenced by internal phenomena that rotate around a single axis.
- The supporting structure is called a stator. As the speed of rotation increases the amplitude of vibration often passes through a maximum that is called a critical speed.
- This amplitude is commonly excited by unbalance of the rotating structure everyday examples include engine balance.
- If the amplitude of vibration at these critical speeds is excessive then catastrophic failure occurs.
- This is the chief concern of engineers who design large rotors.



General equation of motion:

$$M\ddot{q}(t) + (C + G)\dot{q}(t) + (K + N)q(t) = f(t)$$

where:

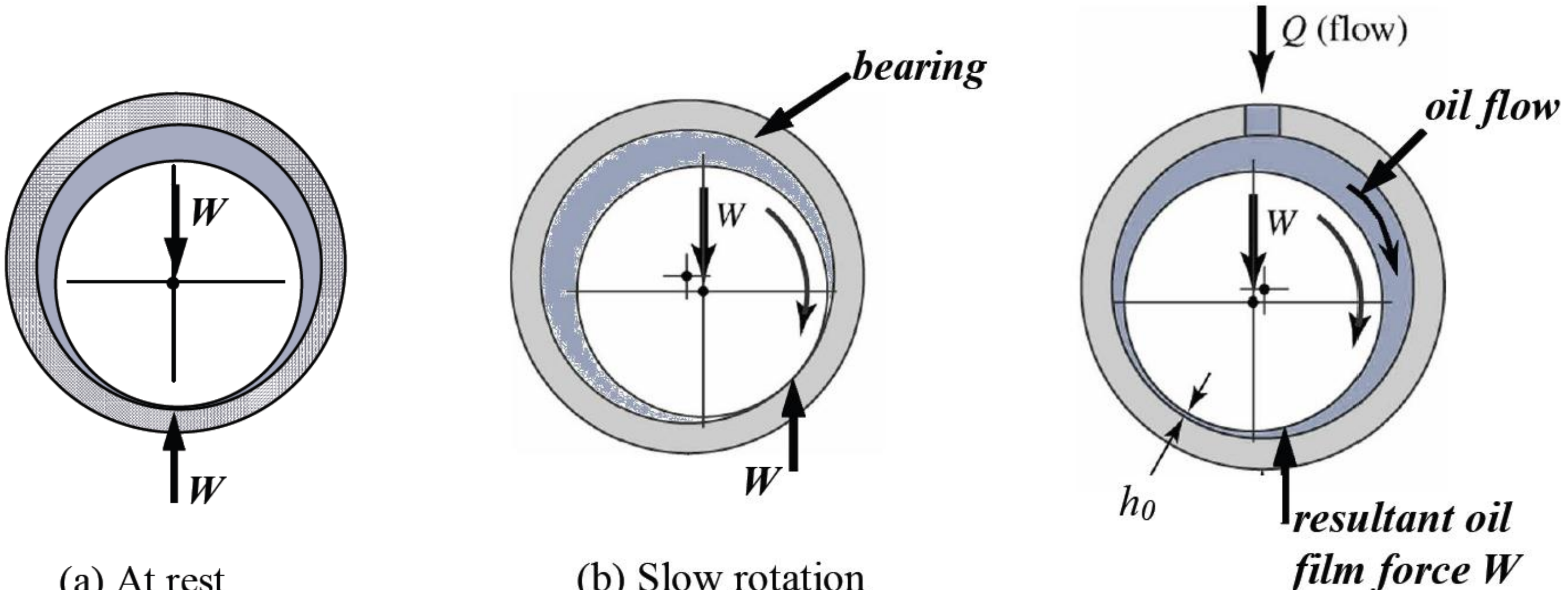


M : Symmetric Mass matrix
C : Symmetric damping matrix
G : Skew-symmetric gyroscopic matrix
K : Symmetric bearing or seal stiffness matrix
N : Gyroscopic matrix of deflection.
q : Generalized coordinates of the rotor in inertial coordinates
f : Forcing function.

- The gyroscopic matrix G is proportional to spin speed Ω. The general solution to the above equation involves complex eigenvectors which are spin speed dependent.

- Rotordynamic system of equations are the off-diagonal terms of stiffness, damping, and mass. These terms are called cross-coupled stiffness, cross-coupled damping, and cross-coupled mass

Hydrodynamic Bearings:



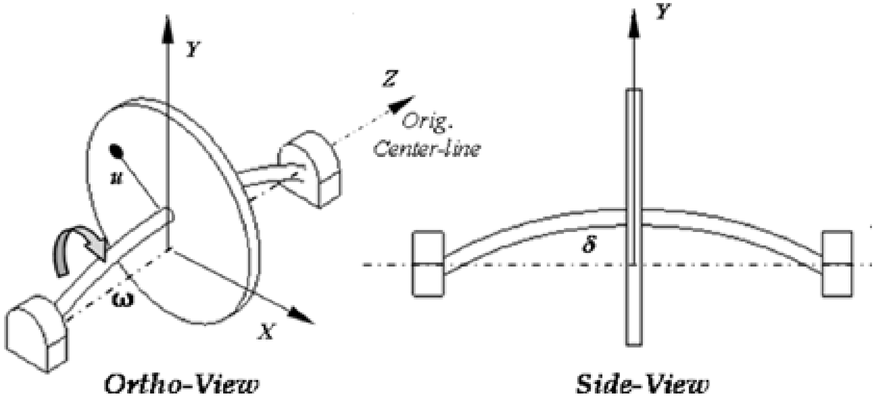
(a) At rest

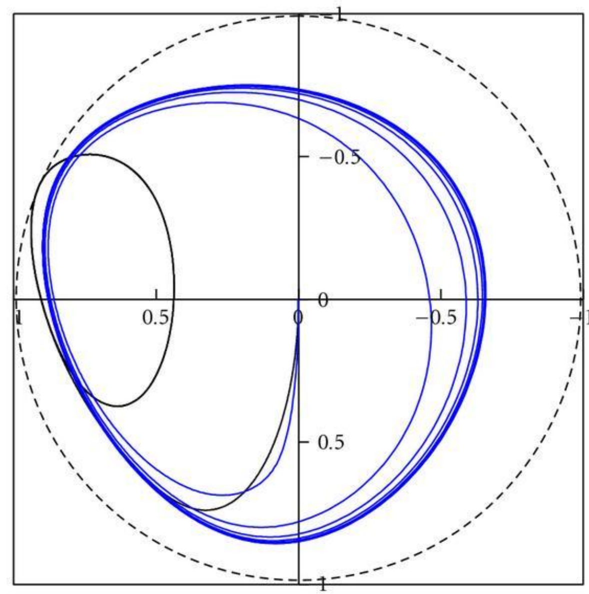
(b) Slow rotation
(Boundary lubrication)

(c) Fast rotation
(Hydrodynamic lubrication)

Whirling:

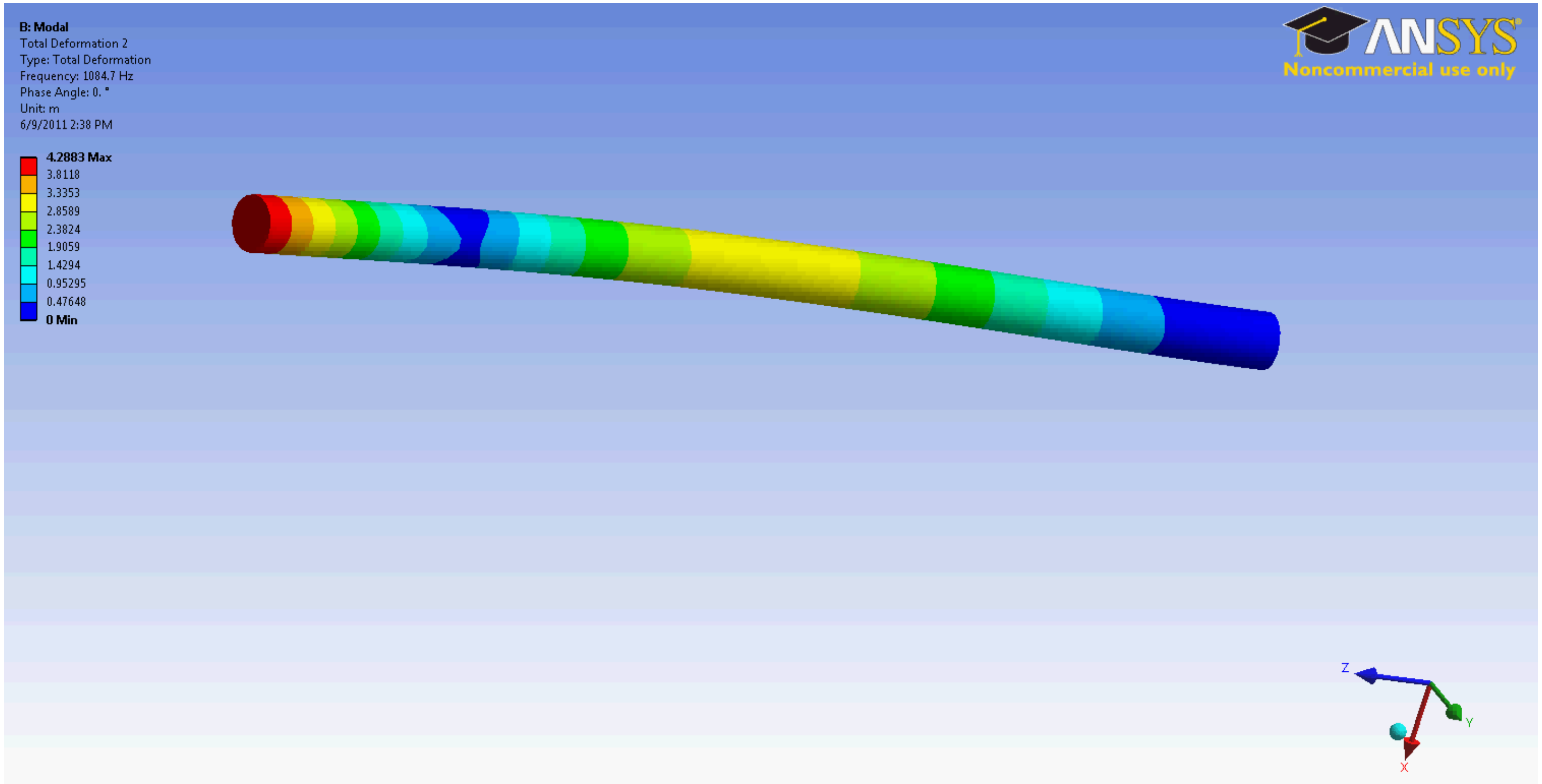
- Whirling is defined as a rotation rapidly about a center or an axis.



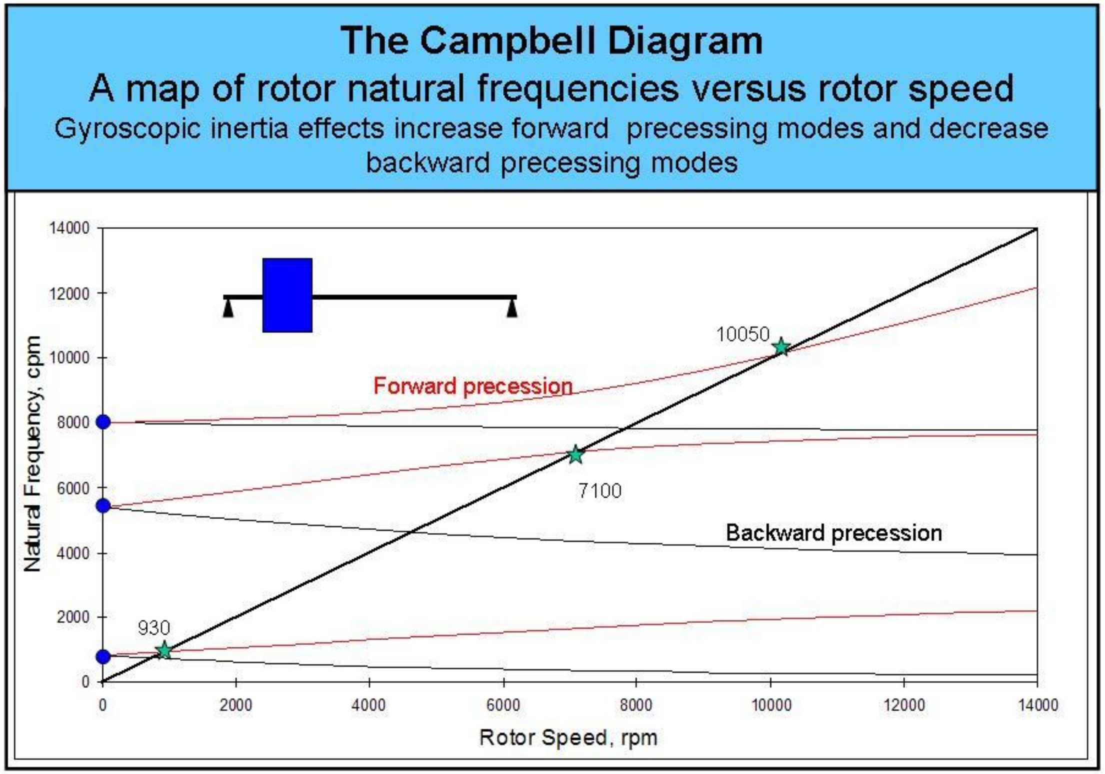


1. What is backward whirling ?
2. What is forward whirling?

Whirling: Example



Campbell diagram:

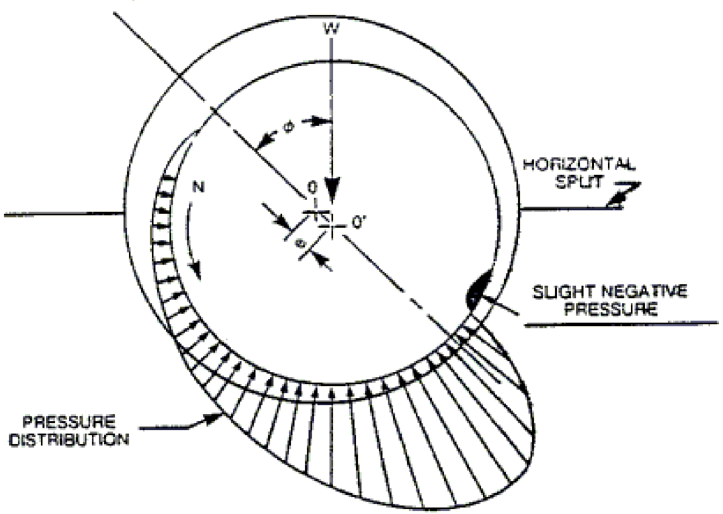


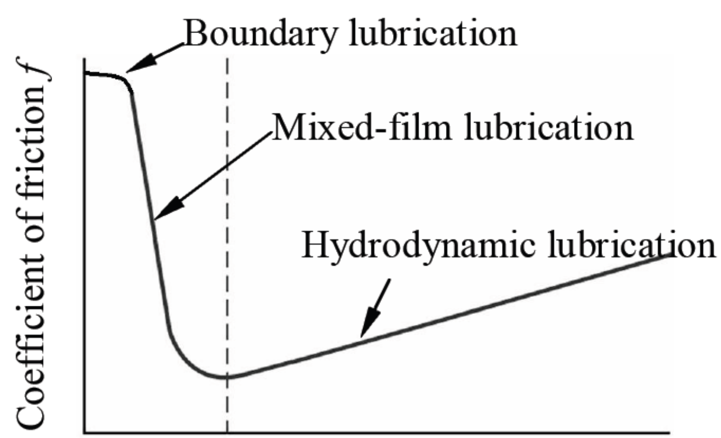
1. What is safety criteria?

Sommerfeld number:

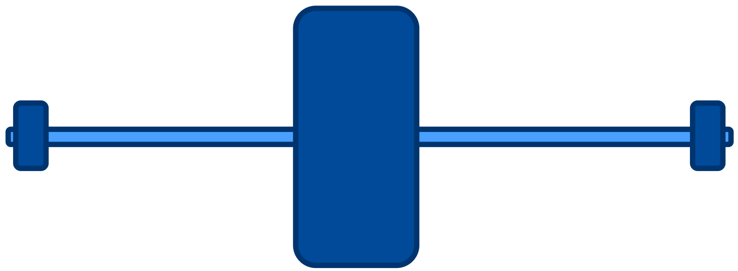
$$S = \left(\frac{r}{c}\right)^2 \frac{\mu.n.D.L}{W}$$

$W = total\ weight$
$r = shaft\ diameter$
$c = clearance$
$n = rotational\ speed$
$\mu = viscosity$
$D = shaft\ diameter$
$L = bearing\ length$





Example:



Shaft Diameter, D: 4 mm
Bearing length, L: 8 mm
Diametric Clearance, C : 1mm
Lubricant : Water
Viscosity : 8.9x 10^-4 Pa.s
Density : 1000kg/m^3
Rotational Speed : 3000 RPM
Weight of rotor per bearing : 1.77 N

$$S = \frac{\mu N \pi D L}{W}\left(\frac{R}{C}\right)^2$$

$$S = \frac{3.14}{1.77} \times 8.9 \times 10^{-4} \times (3000/60) \times 4 \times 8 \times 10^{-6}\left(\frac{2}{0.5}\right)^2$$

$$\mathrm{S} = 4.02 \times 10^{-5} \; (very\ low)$$

$$\epsilon \geq 0.99 \; (very\ high)\ (Predicted\ \ based\ on\ Raimondi\ and\ \ boyd\ chart\ of\ bearing)$$

$$h_{min} = C(1 - \epsilon) = 0.5(1 - 0.99) = 5 \times 10^{-3} mm\ (very\ low)$$

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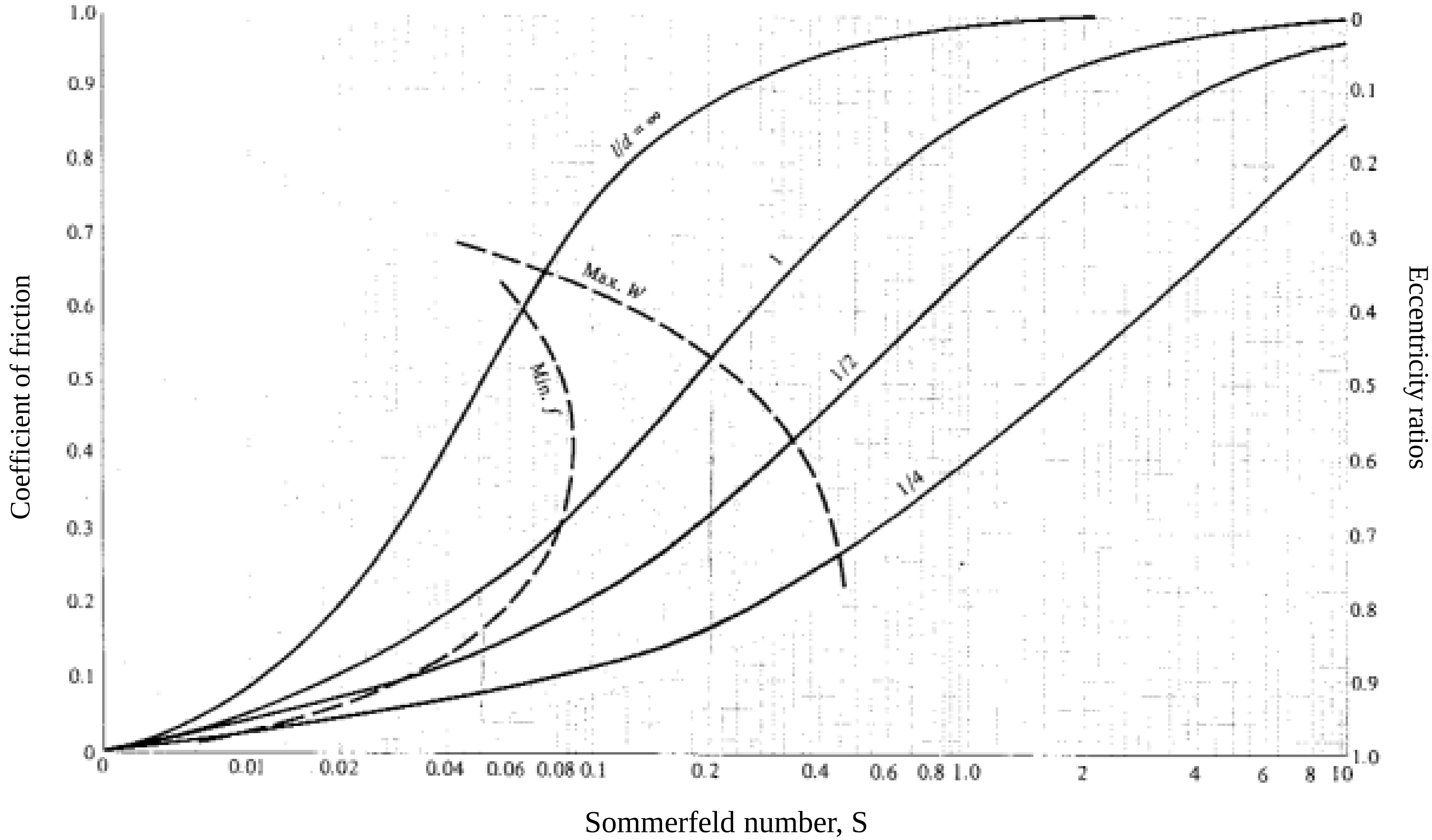
Raimondi and boyd chart:



Software available for Rotordynamics:

- MADYN 2000
- Bond Graph
- Dyrobes
- RIMAP
- XLRotor
- iSTRDYN
- ARMD
- XLTRC2
- ComboRotor
- Dynamics R4
- MESWIR
- RoDAP
- ROTORINSA

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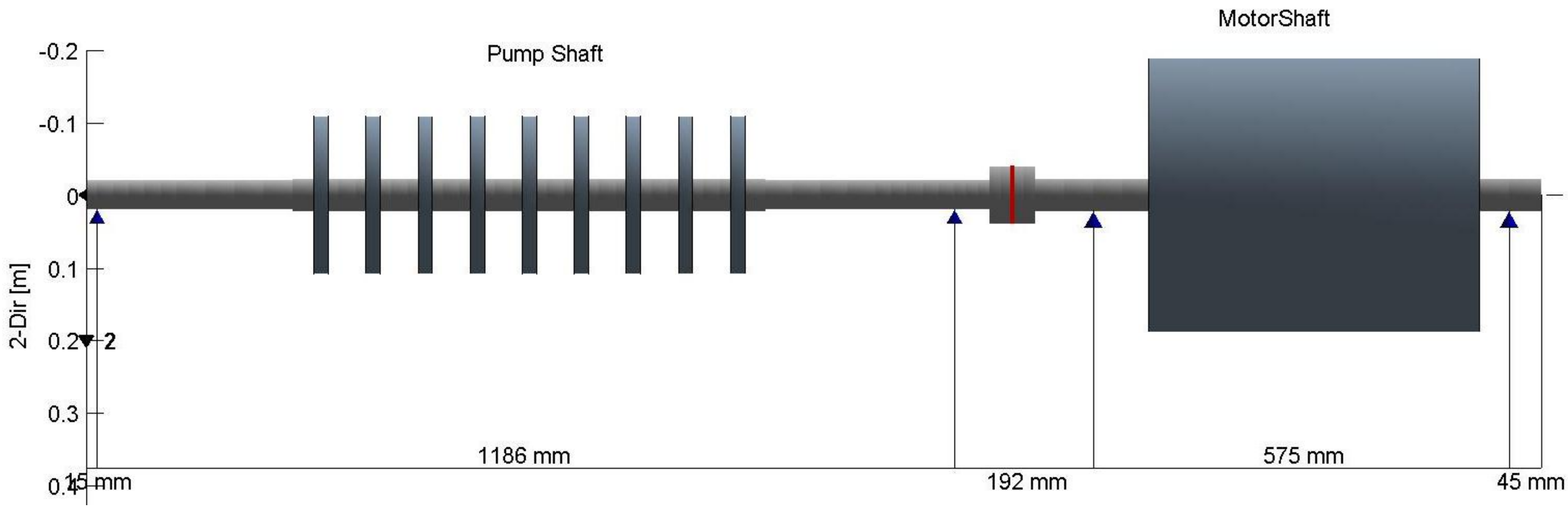
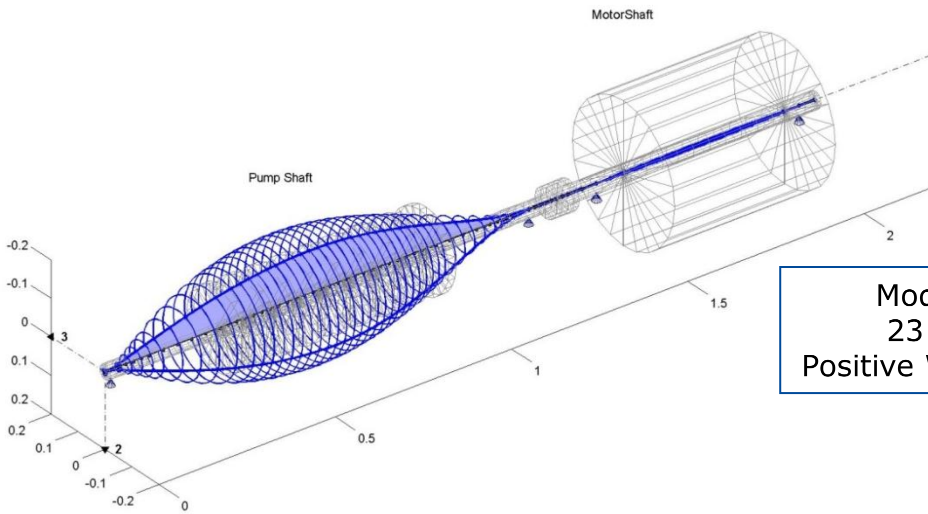
MADYN:

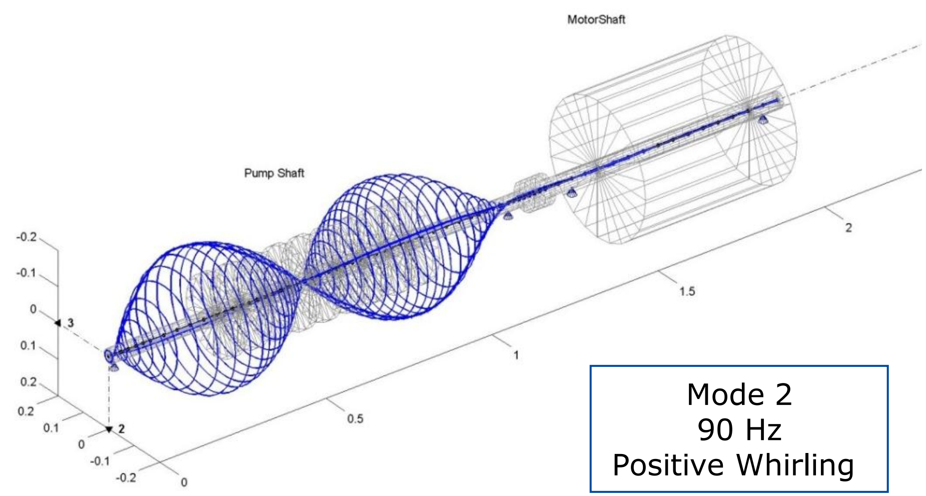


MADYN:

Operating Frequency
48.33 Hz

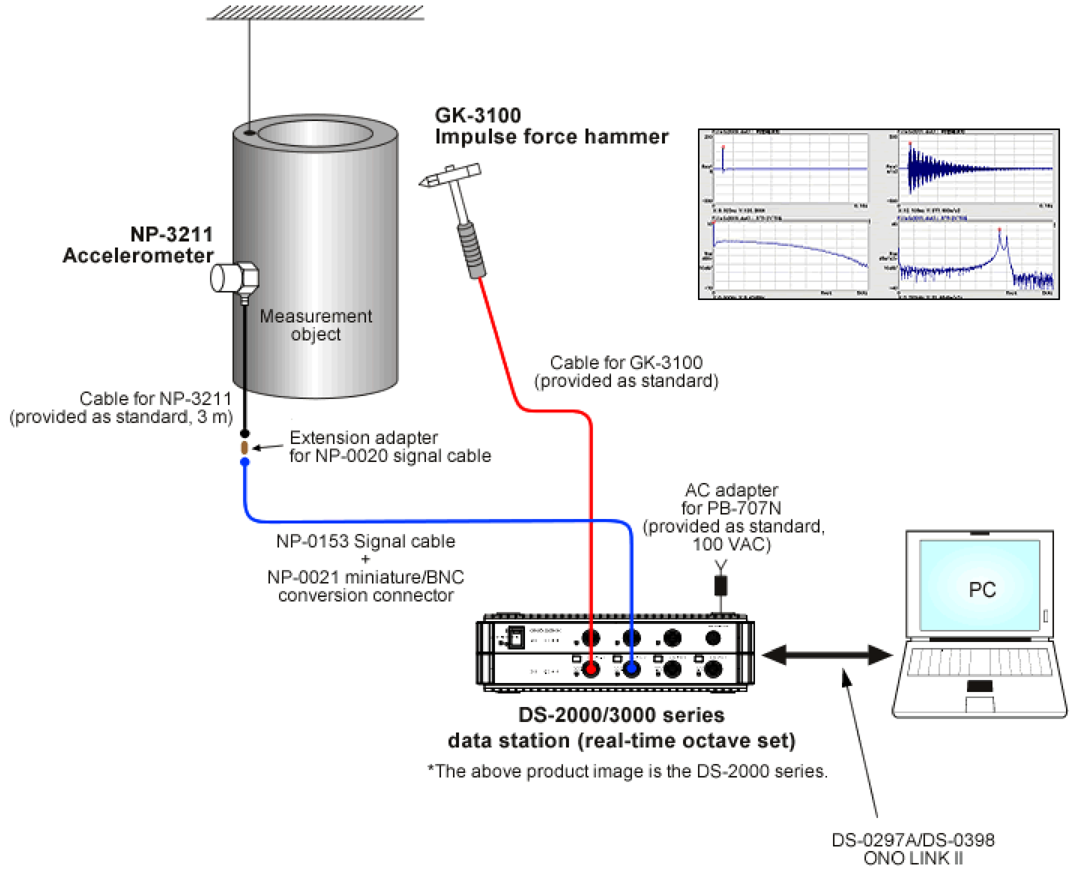


Mode 1
23 Hz
Positive Whirling



Mode 2
90 Hz
Positive Whirling

Vibration Testing:



Measurement of natural frequency and damping ratio by Hammer Testing

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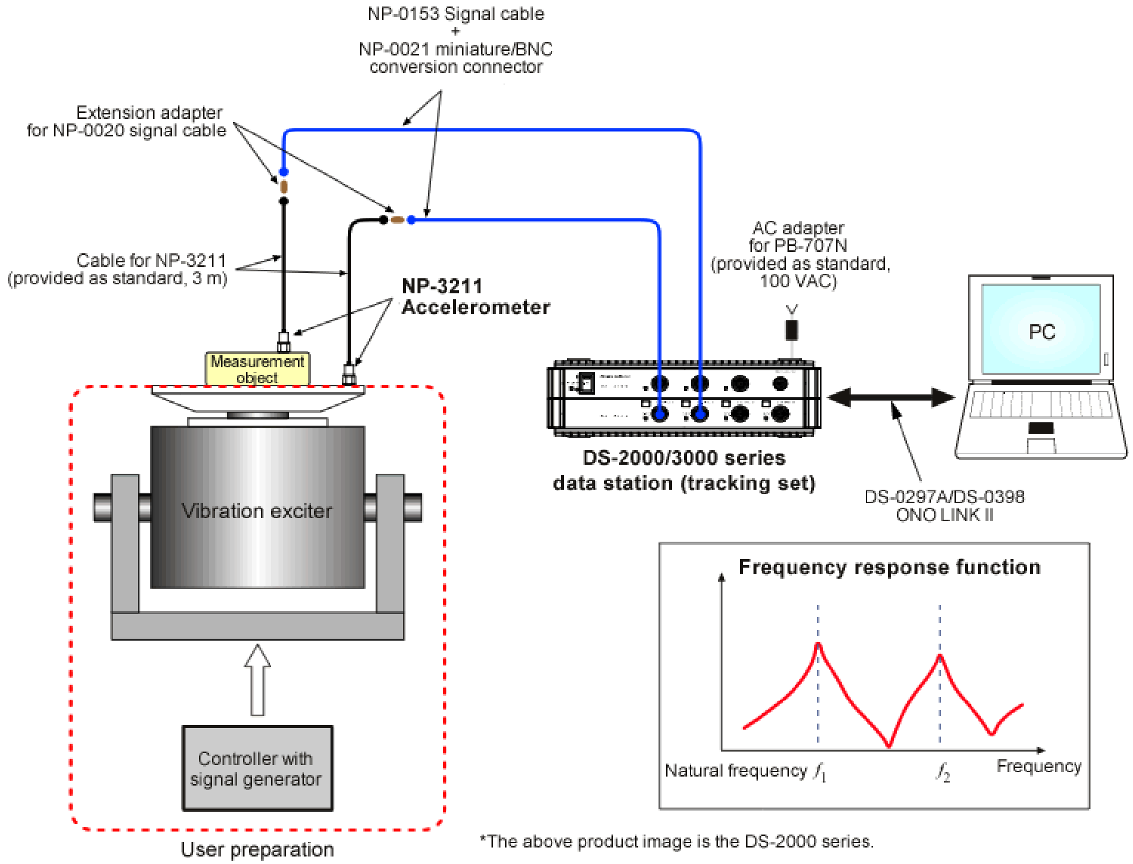
Vibration Testing:



Measurement of natural frequency using exciter controller

Thank You