Bemerkung zu (V21):

$$\ldots = \sum_m \int d^3r \int d^3r' \, \hat{\psi}_m^+(\vec{r}) \frac{1}{(2\pi)^3} \frac{\hbar}{i} \vec{\nabla}_{\vec{r}} \underbrace{\int d^3p \, e^{i\vec{p}\cdot(\vec{r}-\vec{r}')/\hbar}}_{(2\pi)^3 \delta(\vec{r}-\vec{r}')} \times \hat{\psi}_m(\vec{r}') \quad (*)$$

$$= \frac{\hbar}{i} \sum_m \int d^3r \int d^3r' \, \hat{\psi}_m^+(\vec{r}) \, \vec{\nabla}_{\vec{r}} \delta(\vec{r}-\vec{r}') \, \hat{\psi}_m(\vec{r}')$$

$\rightarrow$ Ableitung der δ-Funktion

$$\int_{-\infty}^{\infty} dx \, f(x) \frac{d}{dx} \delta(x-x')$$

$$= \underbrace{f(x)\,\delta(x-x')\Big|_{-\infty}^{\infty}}_{0 \text{ falls } f(x) \to 0 \text{ für } x \to \pm\infty} - \int_{-\infty}^{\infty} dx \, f'(x)\,\delta(x-x') = -f'(x')$$

$$= -\frac{\hbar}{i} \sum_m \int d^3r' \left(\vec{\nabla}_{\vec{r}'} \hat{\psi}_m^+(\vec{r}') \right) \hat{\psi}_m(\vec{r}')$$

$$= \sum_m \int d^3r \left(-\frac{\hbar}{i} \vec{\nabla}_{\vec{r}} \hat{\psi}_m^+(\vec{r}) \right) \hat{\psi}_m(\vec{r})$$

Andererseits hätte ich für (*) auch schreiben können

$$\sum_m \int d^3r \int d^3r' \, \hat{\psi}_m^+(\vec{r}) \frac{1}{(2\pi)^3} \left(-\frac{\hbar}{i} \vec{\nabla}_{\vec{r}'} \underbrace{\int d^3p \, e^{i\vec{p}\cdot(\vec{r}-\vec{r}')/\hbar}}_{(2\pi)^3 \delta(\vec{r}-\vec{r}')} \right) \times \hat{\psi}_m(\vec{r}')$$

$$= -\frac{\hbar}{i}\sum_m \int d^3r \int d^3r' \, \hat{\psi}_m^+(\vec{r}) \left[\vec{\nabla}_{\vec{r}'} \delta(\vec{r}-\vec{r}')\right] \hat{\psi}_m(\vec{r}')$$

nun: $\displaystyle\int_{-\infty}^{\infty} dx' \left(\frac{d}{dx'}\delta(x-x')\right) f(x')$

$$= \underbrace{\delta(x-x') f(x') \Big|_{x'=-\infty}^{\infty}}_{\text{0 wie oben}} - \int_{-\infty}^{\infty} dx' \, \delta(x-x') f'(x')$$

$$= -f'(x)$$

$$= +\frac{\hbar}{i}\sum_m \int d^3r \, \hat{\psi}_m^+(\vec{r}) \, \vec{\nabla}_{\vec{r}} \hat{\psi}_m(\vec{r}) \quad \text{wie (V21)}.$$