Advanced Quantum Theory
WS 10/11

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Problem Set 9 (due 16.12.2010)

(9.1) Excitons *(4 points)*

Show (V94) of the lecture notes, i.e.,

$$\begin{aligned}\hat{H}_{\mathrm{el-d}}|\tilde{\phi}^-\rangle &= -\sum_{k_1\cdots k_4}\{W^{k_1k_2|k_3k_4}_{LV|VL} - W^{k_2k_1|k_3k_4}_{VL|VL}\}\sum_{kk'} c_{kk'}\hat{a}^\dagger_{k_1}\hat{a}_{k_4}\hat{d}^\dagger_{k_3}\hat{d}_{k_2}\hat{a}^\dagger_k\hat{d}^\dagger_{k'}|\phi^-_V\rangle \\ &= -\sum_{k_1\cdots k_4}\{W^{k_1k_4|k_2k_3}_{LV|VL} - W^{k_4k_1|k_2k_3}_{VL|VL}\}\, c_{k_3k_4}\hat{a}^\dagger_{k_1}\hat{d}^\dagger_{k_2}|\phi^-_V\rangle.\end{aligned}$$

(9.2) Bosonic and fermionic operators *(3 points)*

Let $\hat{a}^{(\dagger)}$, $\hat{b}^{(\dagger)}$ be fermionic and bosonic annihilation (creation) operators, respectively, and α a c-number. Show that

$$\mathrm{e}^{-\alpha\hat{b}^\dagger}\;\hat{b}\,\mathrm{e}^{\alpha\hat{b}^\dagger} = \hat{b} + \alpha$$

but

$$\mathrm{e}^{-\alpha\hat{a}^\dagger}\;\hat{a}\,\mathrm{e}^{\alpha\hat{a}^\dagger} = \hat{a} - \alpha^2\hat{a}^\dagger + \alpha(\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}).$$

(9.3) Cooper pairs. *(3 points)*

Show that the following argumentation is wrong: Two electrons in a spin singlet state have total spin zero and thus form a quasi particle, which is a boson. Hence, we can make a Bose condensate of pairs of electrons!

Hint: Consider the field operator for such a *Cooper pair*

$$\hat{\varphi}^\dagger(\mathbf{R}) = \int \mathrm{d}^3r\,\varphi(\mathbf{r})\hat{\psi}^\dagger_\uparrow(\mathbf{R}+\mathbf{r}/2)\hat{\psi}^\dagger_\downarrow(\mathbf{R}-\mathbf{r}/2)$$

which creates a spin singlet state of two electrons, one at position $\mathbf{R}+\mathbf{r}/2$ with spin up and one at position $\mathbf{R}-\mathbf{r}/2$ with spin down. $\varphi(\mathbf{r})$ is the spatial part of the spin-singlet wave function.

Remark: Although the above statement is wrong, Cooper pairs play a crucial role in the theory of superconductivity.