

Problem Set 8 (due 06.12.2011)

Questions

- (Q1) What mathematically is the removal of degeneracies manifests itself physically as the _____ of energy _____.
- (Q2) What is the physical origin of the removal of degeneracies?
- (Q3) What does *fine structure* mean?

(8.1) A carbon configuration

(5 points)

Let us consider the excited-state carbon configuration

$$(1s)^2 (2s)^2 (2p) (3d).$$

- (i) Show that the possible terms ^{2S+1}L are

$$^1P, ^3P, ^1D, ^3D, ^1F, ^3F$$

and determine their degeneracies.

- (ii) Determine the fine-structure multiplet $^{2S+1}L_J$ for the 3P -term above and specify the degeneracy of each fine-structure term.

(8.2) Transformation of annihilation and creation operators

(3 points)

Show that the annihilation operators $\hat{\psi}_m(\mathbf{r})$ and \hat{a}_k obey

$$\hat{\psi}_m(\mathbf{r}) = \sum_k \hat{a}_k \langle \mathbf{r}m | k \rangle.$$

(8.3) Commutators and anti-commutators

(2 points)

Show that for arbitrary operators $\hat{A}, \hat{B}, \hat{C}$

$$[\hat{A}\hat{B}, \hat{C}]_{\mp} = \hat{A}[\hat{B}, \hat{C}]_{\mp} \pm [\hat{A}, \hat{C}]_{\mp} \hat{B},$$

$$[\hat{A}\hat{B}, \hat{C}]_{\pm} = \hat{A}[\hat{B}, \hat{C}]_{\pm} \mp [\hat{A}, \hat{C}]_{\pm} \hat{B}$$

where on the right hand sides one can either choose the upper signs or the lower signs.