

Problem Set 7 (due Monday, 03.12.2012 in the lecture)

Questions

- (Q1) Which atom is bigger, hydrogen or uranium?
- (Q2) Which element has the highest, which one has the lowest ionization potential, and why?
- (Q3) Why are M_L and M_S no good quantum numbers anymore when spin-orbit coupling is taken into account, while L and S are?

(7.1) Total orbital momentum (3 points)

Consider the total orbital angular momentum of the two electrons in He,

$$\hat{\mathbf{L}} = \hat{\mathbf{l}}_1 + \hat{\mathbf{l}}_2.$$

Show that L and M_L are “good quantum numbers” (spin-orbit coupling neglected).

Hint: Show that

$$\left[\hat{\mathbf{L}}, \frac{1}{|\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2|} \right] = \mathbf{0},$$

i.e., any component of $\hat{\mathbf{L}}$ commutes with $\frac{1}{|\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2|}$.

*Why is this sufficient to prove that L and M_L are “good quantum numbers”?

(7.2) A carbon configuration (4 points)

Let us consider the excited-state carbon configuration

$$(1s)^2 (2s)^2 (2p) (3d).$$

- (i) Show that the possible terms ^{2S+1}L are

$$^1P, ^3P, ^1D, ^3D, ^1F, ^3F$$

and determine their degeneracies.

- (ii) Determine the fine-structure multiplet $^{2S+1}L_J$ for the 3P -term above and specify the degeneracy of each fine-structure term.

(7.3) Transformation of annihilation and creation operators (3 points)

Show that the annihilation operators $\hat{\psi}_m(\mathbf{r})$ and \hat{a}_k obey

$$\hat{\psi}_m(\mathbf{r}) = \sum_k \hat{a}_k \langle \mathbf{r}m | k \rangle.$$