Advanced Quantum Theory
WS 10/11

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Problem Set 7 (due 02.12.2010)

(7.1) Morse potential *(3 points)*

The Morse potential introduced in the lecture reads

$$V_{\rm M}(R) = D_{\rm e}\left[{\rm e}^{-2\alpha(R-R_0)} - 2\,{\rm e}^{-\alpha(R-R_0)}\right].$$

(i) Plot $V_{\rm M}(R)$ for the H_2-values $D_{\rm e} = 4.75\,{\rm eV}$, $R_0 = 0.742\,\text{Å}$, and $\alpha R_0 = 1.44$.

(ii) How is α related to $D_{\rm e}$ and k_s, where k_s is the force constant which appeared in the Taylor expansion of $E_s(R)$, i.e.,

$$E_s(R) \simeq E_s(R_0) + \frac{1}{2}k_s(R-R_0)^2.$$

(7.2) Centrifugal distortion *(3 points)*

In the lecture we decoupled rotational and vibrational motion by substituting R_0 for R in the centrifugal term. Instead, consider

$$\left(-\frac{\hbar^2}{2\mu}\frac{{\rm d}^2}{{\rm d}R^2} + V_{\rm eff}(R) - \bar{E}_{s\nu K}\right)\mathcal{F}^s_{\nu K}(R) = 0$$

where $\bar{E}_{s\nu K} = E_{s\nu K} - E_s(\infty)$ and

$$V_{\rm eff}(R) = V_{\rm M}(R) + \frac{\hbar^2}{2\mu}\frac{K(K+1)}{R^2}$$

with $V_{\rm M}(R)$ as given in problem (7.1). Show that the equilibrium internuclear separation is then stretched approximatively to

$$R_1 \simeq R_0 + \frac{\hbar^2}{2\mu}\frac{K(K+1)}{\alpha^2 R_0^3 D_{\rm e}}.$$

(7.3) Repetition: Bosonic creation and annihilation operators *(4 points)*

Let $\hat{b}$ and $\hat{b}^\dagger$ be Bosonic annihilation and creation operators,

$$[\hat{b},\hat{b}^\dagger] = \hat{1}$$

and

$$\hat{n} = \hat{b}^\dagger\hat{b}$$

the occupation number operator, $\hat{n}|n\rangle = n|n\rangle$. Show that

(i) $[\hat{b}^q,\hat{n}] = q\hat{b}^q$ and $[\hat{b}^{\dagger q},\hat{n}] = -q\hat{b}^{\dagger q}$, with q a positive integer, and

(ii) $\hat{b}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{b}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.