Advanced Quantum Theory
WS 11/12

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Problem Set 4 (due 08.11.2011)

(4.1) Mott formula *(5 points)*

The Mott formula [eq. (2.139) in the lecture notes] tells us that the differential cross section for two spin-0 bosons undergoing Coulomb-scattering (with projectile energy E and target at rest in the lab frame) reads in the center-of-mass system

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{Z^2e^2}{4E}\right)^2\left[\frac{1}{\sin^4\theta/2}+\frac{1}{\cos^4\theta/2}+\frac{2\cos(\gamma\ln\tan^2\theta/2)}{\sin^2\theta/2\,\cos^2\theta/2}\right] \quad (1)$$

($\gamma = \frac{Z^2e^2}{\hbar}\sqrt{\frac{\mu}{2E}}$, with reduced mass μ) but for two spin-1/2-fermions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{Z^2e^2}{4E}\right)^2\left[\frac{1}{\sin^4\theta/2}+\frac{1}{\cos^4\theta/2}-\frac{\cos(\gamma\ln\tan^2\theta/2)}{\sin^2\theta/2\,\cos^2\theta/2}\right]. \quad (2)$$

(i) Assuming $E = 10\,\mathrm{MeV}$, plot for the scattering of (a) two ^{12}O-nuclei, (b) two ^{13}O-nuclei, and (c) a ^{12}O-nucleus on a ^{13}O-nucleus $\mathrm{d}\sigma/\mathrm{d}\Omega$ logarithmically vs θ.

(ii) Where does the difference of a factor 2 in the interference term comes from?

(4.2) Antisymmetrization operator *(5 points)*

(i) Let the operator

$$\hat{A}^- = \frac{1}{\sqrt{N!}}\sum_P(-1)^P\hat{P}$$

(notation explained in the lecture) be applied to an arbitrary N-particle state $|\phi\rangle$, i.e.,

$$|\psi\rangle = \hat{A}^-|\phi\rangle.$$

Subsequently, an arbitrary permutation operator $\hat{Q}$ is applied to $|\psi\rangle$. Show

$$\hat{Q}|\psi\rangle = (-1)^Q|\psi\rangle,$$

which means that $\hat{A}^-$ is indeed the antisymmetrization operator.

(ii) Show that

$$\hat{A}^-\hat{A}^- = \sqrt{N!}\,\hat{A}^-.$$

Hint: Use the fact that permutations form a group.

(iii) Consider the product of three single-particle states

$$|\phi\rangle = |1s+\rangle|1s-\rangle|2s+\rangle$$

(where $+$, $-$ indicate $m_s = \pm 1/2$). Write down explicitly $|\psi\rangle = \hat{A}^-|\phi\rangle$.