

Problem Set 3 (due Monday, 04.11.2013 in the lecture)

QUESTIONS

- (Q1) What is the probability current density \mathbf{j} of a particle (i) in a bound and (ii) in a plane-wave state?
- (Q2) What is the Born approximation? What are the requirements for it to be applicable?
- (Q3) What is a Green's function, and what's it good for?

(3.1) YUKAWA

(2 points)

The differential cross section in the first Born approximation

$$\frac{d\sigma}{d\Omega} = \frac{4m^2\alpha^2}{\hbar^4(\mu^2 + 4k^2 \sin^2 \theta/2)^2}$$

for the Yukawa potential $V(r) = (\alpha e^{-\mu r})/r$ has been derived in the lecture. Determine the total cross section σ .

(3.2) GAUSSIAN POTENTIAL

(5 points)

Show that the differential cross section in the first Born approximation for the Gaussian potential $V(r) = V_0 e^{-r^2/r_0^2}$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{\pi r_0^2}{4} \left(\frac{m V_0 r_0^2}{\hbar^2} \right)^2 e^{-q^2 r_0^2 / 2}$$

where $q^2 = 2k^2(1 - \cos \theta)$ is the modulus square of the momentum transfer $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$, as introduced in the lecture. How does $d\sigma/d\Omega$ change as a function of θ with increasing energy of the incident particles?

(3.3) GREEN FUNCTION

(3 points)

In the lecture we introduced the Green function $G^0(\mathbf{r})$ which fulfills

$$(\nabla^2 + k^2)G^0(\mathbf{r}) = \delta(\mathbf{r}). \quad (1)$$

Show that the Fourier-transform of $G^0(\mathbf{r})$ is given by

$$G^0(\mathbf{q}) = \frac{1}{(2\pi)^{3/2}(k^2 - q^2)}.$$

Hint: Do not use the explicit expression for $G^0(\mathbf{r})$ we derived in the lecture but simply Fourier-transform Eq. (1)!