Advanced Quantum Theory
WS 12/13

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Problem Set 3 (due Monday, 05.11.2012 in the lecture)

(3.1) Partial wave expansion *(3 points)*

In the lecture, we made the ansatz

$$\psi_{\mathbf{k}} \stackrel{r\to\infty}{\longrightarrow} \sum_{\ell=0}^{\infty} A_\ell \frac{\mathrm{e}^{\mathrm{i}(kr-\ell\pi/2+\delta_\ell)} - \mathrm{e}^{-\mathrm{i}(kr-\ell\pi/2+\delta_\ell)}}{r} P_\ell(\cos\theta)$$

and determined the coefficients (how?) $A_\ell = (2\ell+1)\mathrm{e}^{\mathrm{i}(\ell\pi/2+\delta_\ell)}/(2\mathrm{i}k)$. Show that

$$\psi_{\mathbf{k}} \stackrel{r\to\infty}{\longrightarrow} \mathrm{e}^{\mathrm{i}kz} + \frac{\mathrm{e}^{\mathrm{i}kr}}{r} \underbrace{\sum_{\ell=0}^{\infty} (2\ell+1)\frac{\mathrm{e}^{2\mathrm{i}\delta_\ell}-1}{2\mathrm{i}k} P_\ell(\cos\theta)}_{f_k(\theta)}$$

follows.

(3.2) s-wave scattering *(4 points)*

In the lecture, we considered a partial wave expansion of the scattering wave function and the scattering amplitude. If, in these expansions, only the term with $\ell = 0$ contributes significantly, the scattering process is called *s-wave scattering*.

(i) What is the requirement for s-wave scattering?

(ii) Determine the s-wave scattering cross section σ_0 for the potential

$$V(r) = \begin{cases} -V_0 & \text{for} \quad r < r_0 \\ 0 & \text{for} \quad r \geq r_0 \end{cases} \tag{1}$$

where $V_0 < 0$ (repulsive) or $V_0 > 0$ (attractive).

(iii) Show that in the low-energy limit $kr_0 \ll 1$ and for "weak" potentials ($k_0 r_0 \ll 1$, $\hbar k_0 = \sqrt{2m|V_0|}$) the cross section

$$\sigma_0 \simeq \frac{4\pi r_0^2}{9}\left(\frac{2mV_0 r_0^2}{\hbar^2}\right)^2 \tag{2}$$

is obtained.

(3.3) Born again *(3 points)*

(i) Determine for the potential (1) the cross section σ in first Born approximation.

(ii) Show that in the low-energy limit $kr_0 \ll 1$ the same expression as in (2) is obtained.

(iii) Show that in the high-energy limit

$$\sigma \simeq \frac{\pi}{2k^2}\left(\frac{2mV_0 r_0^2}{\hbar^2}\right)^2 \sim \frac{1}{E}. \tag{3}$$