Advanced Quantum Theory
WS 15/16

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Problem Set 2 (due Monday, 26.10.2015 in the lecture)

Questions

(Q1) What is Fermi's Golden Rule, and which assumptions have been made during its derivation?

(Q2) "Laser" stands for "light amplification by *stimulated* emission of radiation".[1] How does Fermi's Golden Rule for stimulated emission look like?

(Q3) What is the probability current density $\boldsymbol{j}$ of a particle (*i*) in a bound and (*ii*) in a plane-wave state?

(Q4) What is the Born approximation? What are the requirements for it to be applicable?

(2.1) Time-evolution operator *(3 points)*

Let

$$\hat{H}(t) = \hat{H}_A(t) + \hat{H}_B(t).$$

Show that a time-evolution operator $\hat{U}(t,t')$ fulfilling

$$\hat{U}(t,t') = \hat{U}_A(t,t') - \frac{\mathrm{i}}{\hbar}\int_{t'}^{t} \mathrm{d}t''\, \hat{U}_A(t,t'')\hat{H}_B(t'')\hat{U}(t'',t')$$

also satisfies the time-dependent Schrödinger equation

$$\mathrm{i}\hbar\partial_t\hat{U}(t,t') = \hat{H}(t)\hat{U}(t,t'),$$

if $\hat{U}_A(t,t')$ obeys "its own" time-dependent Schrödinger equation

$$\mathrm{i}\hbar\partial_t\hat{U}_A(t,t') = \hat{H}_A(t)\hat{U}_A(t,t').$$

* What about

$$\begin{aligned}
\hat{U}(t,t') &= \hat{U}_A(t,t') - \frac{\mathrm{i}}{\hbar}\int_{t'}^{t} \mathrm{d}t''\, \hat{U}(t,t'')\hat{H}_B(t'')\hat{U}_A(t'',t'), \\
\hat{U}(t,t') &= \hat{U}_B(t,t') - \frac{\mathrm{i}}{\hbar}\int_{t'}^{t} \mathrm{d}t''\, \hat{U}_B(t,t'')\hat{H}_A(t'')\hat{U}(t'',t'), \\
\hat{U}(t,t') &= \hat{U}_B(t,t') - \frac{\mathrm{i}}{\hbar}\int_{t'}^{t} \mathrm{d}t''\, \hat{U}(t,t'')\hat{H}_A(t'')\hat{U}_B(t'',t') \quad ?
\end{aligned}$$

cont'd overleaf

[1] Learn this by heart! It is frequently asked (and not known) in defenses (BSc, Master, even PhD).

(2.2) DIMENSION *(2 points)*

Determine the dimension of $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ by considering the first-order Born expression

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{Born I}} = \left|\frac{m}{2\pi\hbar^2}\int \mathrm{d}^3r'\, V(\boldsymbol{r}')\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}'}\right|^2 .$$

(2.3) YUKAWA *(2 points)*

The differential cross section in the first Born approximation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{4m^2\alpha^2}{\hbar^4(\mu^2 + 4k^2\sin^2\theta/2)^2}$$

for the Yukawa potential $V(r) = (\alpha\mathrm{e}^{-\mu r})/r$ has been derived in the lecture. Determine the total cross section σ.

(2.4) GAUSSIAN POTENTIAL *(3 points)*

Show that the differential cross section in the first Born approximation for the Gaussian potential $V(r) = V_0\mathrm{e}^{-r^2/r_0^2}$ is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\pi r_0^2}{4}\left(\frac{mV_0r_0^2}{\hbar^2}\right)^2 \mathrm{e}^{-q^2r_0^2/2}$$

where $q^2 = 2k^2(1-\cos\theta)$ is the modulus square of the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_f - \boldsymbol{k}_i$, as introduced in the lecture. How does $\mathrm{d}\sigma/\mathrm{d}\Omega$ change as a function of θ with increasing energy of the incident particles?