Advanced Quantum Theory

WS 12/13

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Problem Set 2 (due Monday, 29.10.2012 in the lecture)

(2.1) Error estimate for variationally determined eigenenergy *(3 points)*

Let $\hat{H}|n\rangle = E_n|n\rangle$ and $E = \langle\psi|\hat{H}|\psi\rangle$, where the trial state $|\psi\rangle$ is already normalized, $\langle\psi|\psi\rangle = 1$. We define the *error vector*

$$|R\rangle = (\hat{H} - E)|\psi\rangle$$

and assume that E_k is that true eigenvalue of $\hat{H}$ which is closest to E. Prove that

$$E - \Delta \leq E_k \leq E + \Delta,$$

where $\Delta = \sqrt{\langle R|R\rangle}$.

(2.2) Yukawa *(2 points)*

The differential cross section in the first Born approximation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{4m^2\alpha^2}{\hbar^4(\mu^2 + 4k^2\sin^2\theta/2)^2}$$

for the Yukawa potential $V(r) = (\alpha\mathrm{e}^{-\mu r})/r$ has been derived in the lecture. Determine the total cross section σ.

(2.3) Gaussian potential *(3 points)*

Show that the differential cross section in the first Born approximation for the Gaussian potential $V(r) = V_0\mathrm{e}^{-r^2/r_0^2}$ is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\pi r_0^2}{4}\left(\frac{mV_0r_0^2}{\hbar^2}\right)^2 \mathrm{e}^{-q^2r_0^2/2}$$

where $q^2 = 2k^2(1-\cos\theta)$ is the modulus square of the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_f - \boldsymbol{k}_i$, as introduced in the lecture. How does $\mathrm{d}\sigma/\mathrm{d}\Omega$ change as a function of θ with increasing energy of the incident particles?

(2.4) Green function *(2 points)*

In the lecture we introduced the Green function $G^0(\boldsymbol{r})$ which fulfills

$$(\nabla^2 + k^2)G^0(\boldsymbol{r}) = \delta(\boldsymbol{r}). \qquad (1)$$

Show that the Fourier-transform of $G^0(\boldsymbol{r})$ is given by

$$G^0(\boldsymbol{q}) = \frac{1}{(2\pi)^{3/2}(k^2 - q^2)}.$$

Hint: Do not use the explicit expression for $G^0(\boldsymbol{r})$ we derived in the lecture but simply Fourier-transform Eq. (1)!