**Advanced Quantum Theory**
WS 15/16

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# **Problem Set 10** (due Monday, 04.01.2016 in the lecture)

## Questions

(Q1) The Dirac equation is a wave equation for a four-component object, called a "bispinor" $\psi$. Why *four* components?

(Q2) Why must the Dirac matrices $\alpha_i$ and $\beta$ not be space-time-dependent?

(Q3) What is the $g$-factor (aka "gyromagnetic factor" or "Landé factor")?

## (10.1) Dirac $\gamma$-matrices *(3 points)*

Without using a particular representation of the $4 \times 4$-matrices $\beta$ and $\alpha_i$, $i = 1, 2, 3$, but only their anti-commutator relations given in the lecture notes, show that by defining

$$\gamma^0 = \beta, \qquad \gamma^i = \beta\alpha_i \tag{1}$$

these anti-commutator relations boil down to

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}1, \qquad \mu, \nu = 0, 1, 2, 3, \tag{2}$$

where $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the metric tensor and the 1 on the right hand side is the $4 \times 4$ identity matrix.

## (10.2) Repetition: metric tensor *(4 points)*

The square of the distance of two infinitesimally distant points is the square of the *line element* $\mathrm{d}s$ (sum convention),

$$\mathrm{d}s^2 := (\mathrm{d}s)^2 = \mathrm{d}x_\mu\, \mathrm{d}x^\mu = g_{\mu\nu}\, \mathrm{d}x^\nu\, \mathrm{d}x^\mu. \tag{3}$$

Here, $\mathrm{d}s^2$ is, according to exercise (9.4), a scalar, and $g_{\mu\nu}$ is the *metric tensor* which "pulls down" indices:

$$g_{\mu\nu}\mathrm{d}x^\nu = \mathrm{d}x_\mu. \tag{4}$$

It can be shown that $g_{\mu\nu}$ is a (2nd-rank) covariant tensor, i.e.,

$$\bar{g}_{\mu\nu} = \frac{\partial x^\kappa}{\partial \bar{x}^\mu}\frac{\partial x^\sigma}{\partial \bar{x}^\nu}\, g_{\kappa\sigma}. \tag{5}$$

Starting from Cartesian coordinates $x, y, z$ for which $g = \text{diag}(1, 1, 1)$, derive the metric $\bar{g}$ for spherical coordinates $\bar{x} = (r, \theta, \phi)$.

*cont'd overleaf*

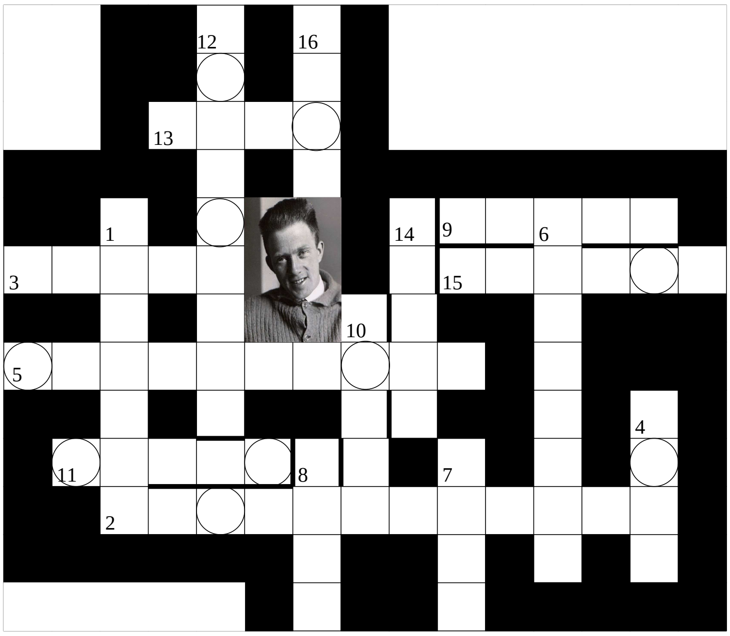
(10.3) IDENTITY INVOLVING PAULI MATRICES AND THREE-VECTORS *(3 points)*

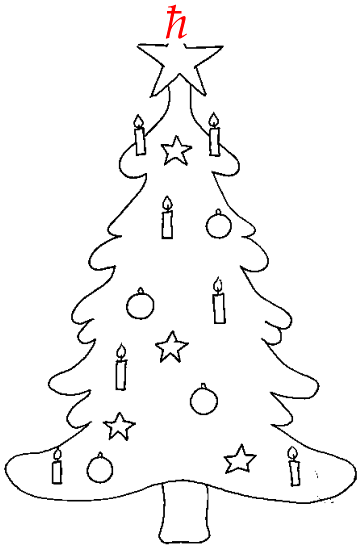
Show the identity used in the lecture

$$(\sigma \cdot \mathbf{a})(\sigma \cdot \mathbf{b}) = \mathbf{a} \cdot \mathbf{b}\, 1 + i\sigma \cdot (\mathbf{a} \times \mathbf{b}).$$

Here, **a** and **b** are arbitrary three-vectors, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices, and the 1 in the first term on the right hand side is the $2 \times 2$ identity matrix.

(10.4) *CHRISTMAS QUIZ




We are looking for names of physicists or mathematicians that appear in the lecture notes. The encircled letters, put together in the right order, give the name of the physicist pictured.

ö=oe (who might this be?).

1. 2nd in WKB
2. Austrian, shared Nobel prize with 3.
3. "The Strangest Man".
4. Solves eq. of 2. for constant force.
5. Doctoral advisor of several others here.
6. Previously known as John W. Strutt.
7. Swiss "crack".
8. Russian "head sail".
9. Estimated number of piano tuners in Chicago.
10. Dipole moment in cgs.
11. Father and son won Nobel prize together.
12. Great mathematician, but mostly known for ridiculous symbol.
13. Grandfather of John Travolta's leading lady in "Grease".
14. Ingenious, sarcastic slugabed whose withering assessment was "Not even wrong!".
15. Founder of SSMTG @ MIT.
16. Was briefly in Rostock and has his rules.