

Seminar 7

Keywords:

Natural line width, symmetries, groups, invariance,
conservation laws, translation/rotation operators

Questions:

1. Explain the physical origin of natural line widths.
2. What is the relation between symmetries and conservation laws?
3. Is the Hamiltonian of a free particle Galilei-invariant?

Assignment 7

(due November 30, 2009)

Inversion operator

7.1 Show that

$$\hat{D}(i) = \exp \left[\frac{i\pi}{2\hbar} \left(\alpha \hat{x}^2 + \frac{1}{\alpha} \hat{p}^2 - 3\hbar \hat{1} \right) \right], \quad (\alpha \neq 0, \text{ real})$$

is a representation of the inversion operation by transforming \hat{x} and \hat{p} with it.

Particle in a homogeneous magnetic field

7.2 The problem of a charged particle (charge q , mass m) in a homogeneous magnetic field $\mathbf{B} = \nabla \times \hat{\mathbf{A}}(\hat{\mathbf{x}})$ (with the vector potential $\hat{\mathbf{A}}(\hat{\mathbf{x}}) = (\mathbf{B} \times \hat{\mathbf{x}})/2$) is also invariant under translations (why?). However, show that $\hat{T}(\boldsymbol{\xi}) = \exp(-i\boldsymbol{\xi} \cdot \hat{\mathbf{p}}/\hbar)$ does not commute with the Hamiltonian \hat{H} . Introduce a proper translation operator that *does* commute with \hat{H} .

Yet another operator

7.3 Find out what the effect of the following operator is:

$$\hat{D}(a) = \exp \left[\frac{ia}{2\hbar} (\hat{\mathbf{p}}\hat{\mathbf{x}} + \hat{\mathbf{x}}\hat{\mathbf{p}}) \right].$$

Free falling

7.4 Solve the Schrödinger equation for a particle of mass m that falls freely under gravity close to the surface of the earth, i.e., $V(z) = mgz$, $z \geq 0$, and bounces back from an elastic plane located at $z = 0$.