Advanced Theoretical Physics
WS 09/10

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Seminar 6

Keywords:

Pictures, Ehrenfest theorem, free wave packets,
time-ordering operator, time-dependent perturbation theory

Questions:

1. Describe a free wave packet in momentum representation and show how momentum and position space representation are related.
2. Why do free wave packets spread and why does the spreading depend on the initial width of the packet?
3. In the lecture we restricted ourselves to one-dimensional free wave packets. How can the results be generalized to three dimensions?

Assignment 6

(due November 23, 2009)

Commutators

6.1 Show that the position operators at two times t_0 and t of (a) a free particle and (b) the harmonic oscillator in the Heisenberg picture fulfill

$$[\hat{x}(t),\hat{x}(t_0)] = \frac{\hbar}{\mathrm{i}}\,\frac{t-t_0}{m}\,\hat{1}, \qquad \text{(free)}$$

$$[\hat{x}(t),\hat{x}(t_0)] = \frac{\hbar}{\mathrm{i}}\,\frac{\sin\omega(t-t_0)}{m\omega}\,\hat{1}, \qquad \text{(harm. osc.)}$$

respectively. Interprete the results.

Free propagator and wave packet

6.2 Show (the steps skipped in the lecture, namely) that

$$U(x,x';t) = \frac{1}{2\pi}\int \exp\left\{\mathrm{i}\left[k(x-x') - \frac{\hbar}{2m}k^2 t\right]\right\}\,\mathrm{d}k = \sqrt{\frac{m}{2\pi\hbar \mathrm{i} t}}\,\exp\left\{\frac{\mathrm{i}m}{2\hbar}\,\frac{(x-x')^2}{t}\right\},$$

and thus for $\phi(x,0) = A\exp[\mathrm{i}k_0 x - x^2/(2a^2)]$

$$\phi(x,t) = \frac{A}{\sqrt{1+\mathrm{i}\frac{\hbar t}{ma^2}}}\exp\left\{-\frac{x^2 - 2\mathrm{i}a^2k_0x + \mathrm{i}\frac{\hbar a^2 k_0^2}{m}t}{2a^2\left(1+\mathrm{i}\frac{\hbar t}{ma^2}\right)}\right\}.$$

Delta potential

6.3 Determine the energy eigenvalue and eigenfunction in position space representation for the one-dimensional problem of a particle of mass m in a potential $V(x) = V_0\,\delta(x)$ with $V_0 > 0$.