Advanced Theoretical Physics
WS 09/10

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Seminar 3

Keywords:

Multiplicity and entropy, 3. law of thermodynamics,
wave-particle dualism, states, operators, observables, commutators

Questions:

1. What are the entropy and the chemical potential of a photon gas?
2. What are the de Broglie wavelengths of (a) Usain Bolt during his 100 m world record, (b) a dust particle moving with 1 mm/s, (c) a thermal neutron at room temperature, (d) an electron accelerated by a potential difference V (in Volts), (e) an electron in the hydrogen groundstate, and (f) a cosmic ray electron?

Assignment 3

(due November 2, 2009)

Momentum space volume

3.1 In the lecture we calculated the multiplicity $\mathcal{N}$ of a classical ideal gas of N indistinguishable particles via the phase space volume occupied by states *up to* energy E,

$$\Omega(E) = V^N \frac{(2\pi m E)^{3N/2}}{(3N/2)!},$$

while for the entropy $S = k_\mathrm{B} \ln \mathcal{N}_N(E)$ actually the multiplicity $\mathcal{N}_N(E)$ refers to states *of* energy E. Show that nevertheless in "logarithmic accuracy" $\mathcal{N}_N(E) = \Omega(E)/(h^3 N!)$ can be used.

Thermodynamics of a spring

3.2 Consider a spring which follows Hooke's law $Y = k(T)X$, with X the elongation, Y the tension, and $k(T)$ the temperature-dependent spring constant. Determine the Helmholtz free energy A, the internal energy U, and the entropy S as a function of X. Neglect thermal expansion.

Scalar product

3.3 Calculate the norms $\|\varphi\| = \sqrt{\langle\varphi|\varphi\rangle}$, $\|\chi\|$, and the scalar product $\langle\varphi|\chi\rangle$ for the vectors

(a) $|\varphi\rangle = (|v_1\rangle + \mathrm{i}|v_2\rangle)/\sqrt{2}, \qquad |\chi\rangle = (|v_1\rangle - \mathrm{i}|v_2\rangle)/\sqrt{2};$
(b) $|\varphi\rangle = \int_a^b |k\rangle \mathrm{e}^{\mathrm{i}k}\, \mathrm{d}k, \qquad |\chi\rangle = \int_a^b |k\rangle \mathrm{e}^{2\mathrm{i}k}\, \mathrm{d}k.$

Useful formulas for operators

3.4 Let $[\hat{L}, [\hat{L}, \hat{M}]] = [\hat{M}, [\hat{L}, \hat{M}]] = 0$. Show that $\mathrm{e}^{\hat{L}+\hat{M}} = \mathrm{e}^{\hat{L}}\mathrm{e}^{\hat{M}}\mathrm{e}^{-[\hat{L},\hat{M}]/2}$.

3.5 Let the operator $\hat{L}$ and its inverse $\hat{L}^{-1}$ depend on a parameter t. Show that

$$\frac{\mathrm{d}\hat{L}^{-1}}{\mathrm{d}t} = -\hat{L}^{-1}\frac{\mathrm{d}\hat{L}}{\mathrm{d}t}\hat{L}^{-1}.$$