

Exam
Advanced Theoretical Physics
WS 09/10

Prof. Dr. Dieter Bauer
Dipl.-Phys. Andrea Sengebusch

Thermodynamics

1. Let an equation of state be given in the form $p = p(T, V)$. Show that

$$p\alpha_p = k\alpha_V, \quad \text{where} \quad \alpha_p = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_V, \quad \alpha_V = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p, \quad k = -V \left(\frac{\partial p}{\partial V} \right)_T.$$

2. Show that the internal energy of a material whose equation of state has the form $p = f(V)T$ [where $f(V)$ is an arbitrary function of V] is independent of the volume, i.e., $(\partial U / \partial V)_T = 0$.

Quantum Theory and Statistics

3. Determine the energy eigenvalues E_n , $n = 0, 1, 2, \dots$ of a particle of mass m in the potential (one-dimensional problem)

$$V(x) = \begin{cases} m\omega^2 x^2 / 2 & \text{for } x \geq 0 \\ +\infty & \text{for } x < 0 \end{cases}$$

where $\omega > 0$ is a real constant.

Hint: Use the well-known result for the one-dimensional harmonic oscillator.

4. An operator \hat{A} commutes with any two of the three components of an angular momentum operator $\hat{\mathbf{J}}$. Show that it then also commutes with the third component.
5. Consider a system of three spin $s = 1/2$ -particles. Let $\hat{s}_z|\pm\rangle = \pm\frac{\hbar}{2}|\pm\rangle$ and $\hat{\mathcal{A}}$ be the anti-symmetrization operator. Calculate $\hat{\mathcal{A}}|u\rangle$, where

$$|u\rangle = \frac{1}{\sqrt{2}}(|+-+\rangle - |-++\rangle).$$

6. The canonical partition function of the photon gas can be written as

$$Z = \left(\prod_{\mathbf{p} \neq 0} \frac{1}{1 - e^{-\beta E_{\mathbf{p}}}} \right)^2,$$

where $E_{\mathbf{p}} = c|\mathbf{p}|$ is the energy of a photon of momentum \mathbf{p} and $\beta = 1/k_B T$. Show that the free energy $F(T, V) = -k_B T \ln Z$ is proportional to $(k_B T)^4$.

Hint: Make use of the fact that $\sum_{\mathbf{p}} \dots$ can be written as an expression which is proportional to $\frac{V}{(2\pi\hbar)^3} \int d^3p \dots$.

(cont'd overleaf)

