

### Problem Set 3

**(3.1) Sum rule.** For the sake of brevity, the Thomas-Reiche-Kuhn sum rule for the oscillator strengths  $f_{ka}^{(N)}$ ,

$$\sum_k f_{ka}^{(N)} = N, \quad f_{ka}^{(N)} = \frac{2m\omega_{ka}}{3\hbar} |\mathbf{R}_{ka}|^2, \quad \mathbf{R}_{ka} = \sum_{i=1}^N \mathbf{r}_{ka}^{(i)}, \quad \mathbf{r}_{ka}^{(i)} = \langle \psi_k | \mathbf{r}_i | \psi_a \rangle$$

was derived only for  $N = 1$ -electron atoms in the lecture. Show that it also holds for  $N$ -electron atoms.

**(3.2) Cascade decay.** An atom has a level scheme as depicted in Fig. 1 below. Because state  $|a\rangle$  and the ground state  $|g\rangle$  have the same parity, the system decays via  $|b\rangle$ , which is a state of opposite parity. Suppose an experiment in which  $|a\rangle$  is instantly populated at time  $t = t_0$  and in which the emission of photons due to the spontaneous decay from  $|b\rangle$  to  $|g\rangle$  (fluorescence) is detected.

(i) Derive the time-dependence of the fluorescence signal in terms of the lifetimes  $\tau_a$ ,  $\tau_b$  of states  $|a\rangle$  and  $|b\rangle$ , respectively.

*Hint:* Write down and solve rate equations, and make use of the fact that the fluorescence signal is proportional to the rate at which atoms return from state  $|b\rangle$  to the ground state  $|g\rangle$ .

(ii) Analyze the limiting cases  $\tau_a \gg \tau_b$ ,  $\tau_a \ll \tau_b$ , and  $\tau_a \simeq \tau_b$ .

(iii) Does the shape of the fluorescence signal change if  $|a\rangle$  has additional decay channels to states other than  $|b\rangle$ ?

(iv) The right-hand-side panel in Fig. 1 shows an actual measurement of the fluorescence signal from dysprosium. Determine  $\tau_a$  and  $\tau_b$ .

*Just for information:* The state  $|a\rangle$  ( $E = 19797.96 \text{ cm}^{-1}$ ) was populated with a sequence of short nanosecond-laser pulses. The  $|b\rangle \rightarrow |g\rangle$ -transition gives rise to fluorescence at 564 nm.

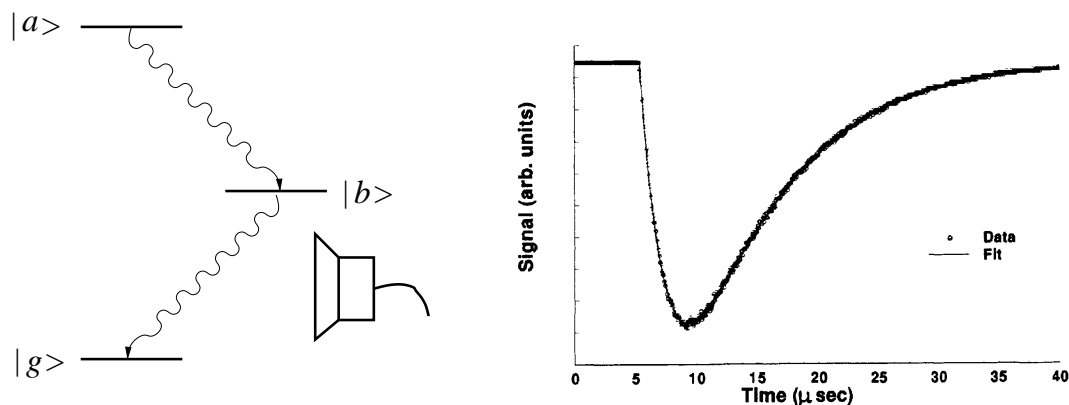


Figure 1: Level scheme (left) and fluorescence signal from Dy for the  $|b\rangle \rightarrow |g\rangle$ -transition as a function of time [right, from Budker et al., Phys. Rev. A **50**, 132 (1994)].