Atoms and Clusters
WS 11/12

Dr. Tatyana Liseykina
Prof. Dr. Karl-Heinz Meiwes-Broer
Prof. Dr. Dieter Bauer

Problem Set 10

Questions

(Q.1) The ideal BEC is a non-interacting quantum gas demonstrating a phase transition. How is this possible?

(Q.2) What is the critical temperature of a BEC and how does it scale with mass and density of the atoms?

Problems

(10.1) **BEC in 2D?**
It is possible to realize a 2D Bose gas by, e.g., trapping He atoms on the surface of another material. (i) Show that in the 2D case Eq. (2.4) from the lecture notes is to be replaced by $\frac{A}{(2\pi)^2}\,\mathrm{d}^2k$ and (2.8) by $g(\epsilon) = \frac{m}{2\pi\hbar^2}$. (ii) Calculate the chemical potential μ for the 2D case and show that it never vanishes and thus a BEC in 2D cannot exist.

(10.2) **Zeeman splitting**
Using the wave functions of the different $S = 2$ states for $M_S = 2, 1, 0, -1, -2$ show that in first order in B_z the magnetic field changes the energy levels by

$$\Delta E = +\frac{1}{2}\mu_B M_S B_z.$$

For the $S = 1$ energy levels show that

$$\Delta E = -\frac{1}{2}\mu_B M_S B_z.$$

Show that this is consistent with the energy level scheme sketched in Fig. 2.3 of the lecture notes.

(10.3) **Harmonically trapped BEC**

(i) Approximating the trapping potential in a magnetic trap by a three-dimensional harmonic oscillator potential

$$V_{\text{trap}} = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$$

in the Gross-Pitaevskii equation (2.60) and ignoring the interaction ($g = 0$), show that the single particle quantum states have energies

$$\epsilon_{n_x n_y n_z} = \hbar\omega(n_x + n_y + n_z + \frac{3}{2}).$$

(ii) Find the total number of quantum states with energies less than ϵ, and thus show that the density of states is

$$g(\epsilon) = \frac{\epsilon^2}{2(\hbar\omega)^3}$$

when ϵ is large.

(iii) Write down the analog of Eq. (2.22) for this density of states, and show that the BEC temperature for N atoms in the trap is of order $T_c \sim N^{1/3}\hbar\omega/k_B$ when N is large.