

Wichtige Integrale

$$(1) \quad \int_0^{\infty} x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}} \quad a > 0 \quad n \in \mathbb{N}_0$$

$$(2) \quad \int_0^{\infty} e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \quad a > 0$$

$$(3) \quad \int_0^{\infty} x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}} \quad a > 0 \quad n \in \mathbb{N}$$

$$(4) \quad \int_0^{\infty} x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{2 a^{n+1}} \quad a > 0 \quad n \in \mathbb{N}_0$$

$$(5) \quad \int_0^{\infty} \frac{x^3}{e^x - 1} \, dx = \frac{\pi^4}{15}$$

$$(6) \quad \int_0^{\infty} \frac{x^4 e^x}{(e^x - 1)^2} \, dx = \frac{4\pi^4}{15}$$

$$(7) \quad \int \sin^2(ax) \, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$(8) \quad \int x \sin^2(ax) \, dx = \frac{x^2}{4} - \frac{x \sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$(9) \quad \int x^2 \sin^2(ax) \, dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin(2ax) - \frac{x}{4a^2} \cos(2ax)$$

$$(10) \quad \int \sin(ax) \cos(ax) \, dx = \frac{1}{2a} \sin^2(ax)$$

$$(11) \quad \int \sin(ax) \sin(bx) \, dx = \frac{\sin((a-b) \cdot x)}{2(a-b)} - \frac{\sin((a+b) \cdot x)}{2(a+b)} \quad |a| \neq |b|$$