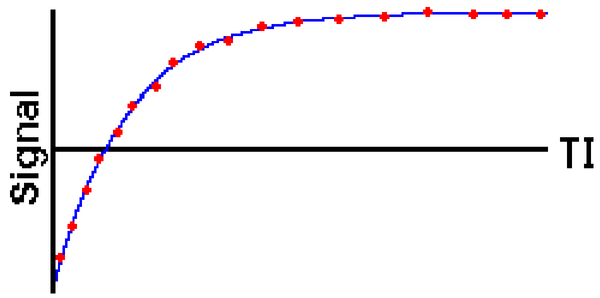
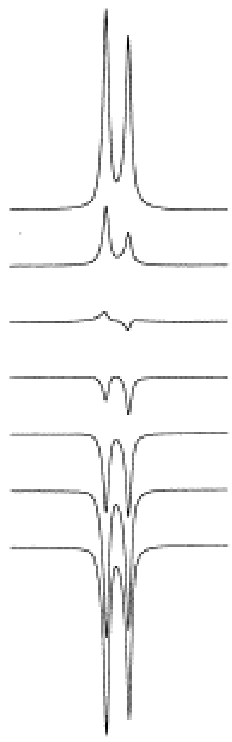
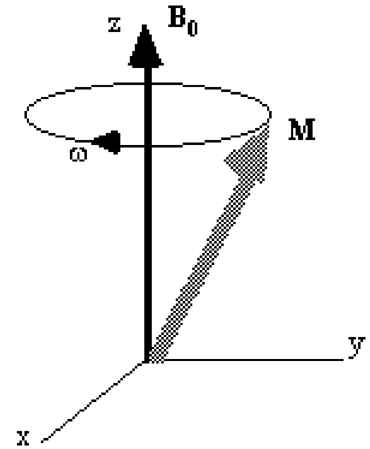
NMR Relaxation





NMR Relaxation



$$\frac{d\vec{M}}{dt} = \gamma\, \vec{M} \times \vec{B}$$

$$\frac{dM_x}{dt} = \gamma\, M_y \times B_0 \qquad \frac{dM_y}{dt} = -\gamma\, M_x \times B_0 \qquad \frac{dM_z}{dt} = 0$$

Lösungen:

$$M_x(t) = M_x(0)\cos\omega_0 t + M_y(0)\sin\omega_0 t$$

$$M_y(t) = -M_x(0)\sin\omega_0 t + M_y(0)\cos\omega_0 t$$

$$M_z(t) = M_z(0)$$

NMR Laborsystem

B_1-Feld senkrecht zu B_0 mit ω_0: $\vec{B}_1(t) = B_1 \cos\omega_0 t\,\vec{i} - B_1 \sin\omega_0 t\,\vec{j}$

$$\frac{dM_x}{dt} = \gamma\left[M_y B_0 + M_z B_1 \sin\omega_0 t\right]$$

$$\frac{dM_y}{dt} = \gamma\left[M_z B_1 \cos\omega_0 t - M_x B_0\right]$$

$$\frac{dM_z}{dt} = \gamma\left[-M_x B_1 \sin\omega_0 t - M_y B_1 \cos\omega_0 t\right]$$

Lösungen:

$$M_x(t) = M_0 \sin\omega_1 t \sin\omega_0 t$$

$$M_y(t) = M_0 \sin\omega_1 t \cos\omega_0 t$$

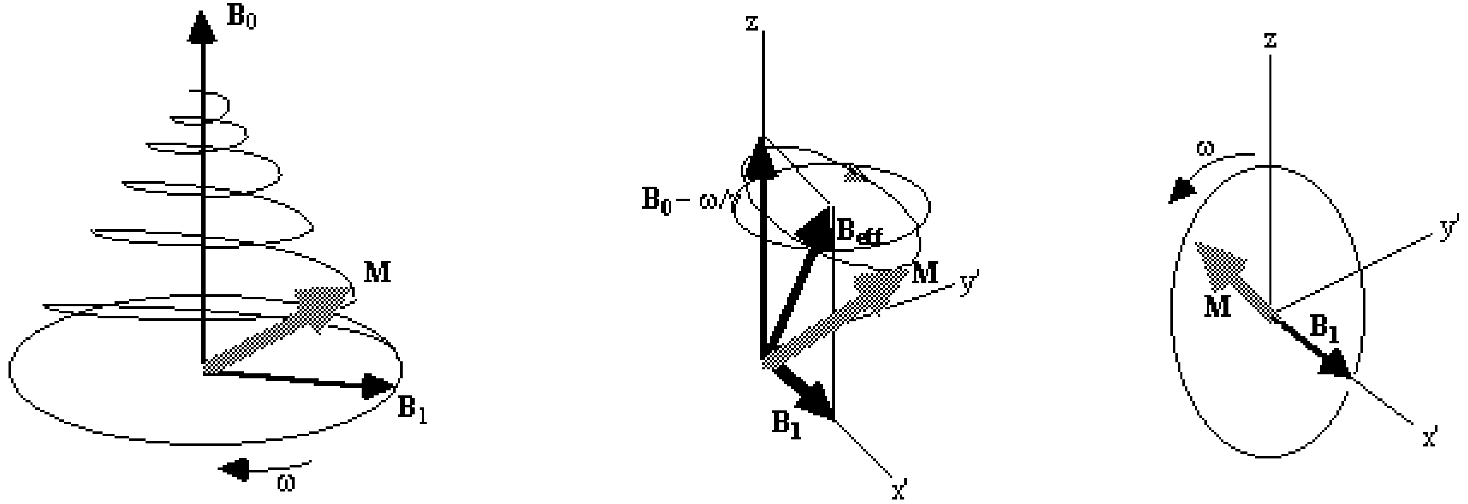
$$M_z(t) = M_0 \cos\omega_0 t$$

NMR Rotierendes Koordinatensystem

$$\frac{d\vec{M}}{dt} = \gamma\, \vec{M} \times \vec{B}_{eff}$$

(i',j',k) Einheitsvektoren in (x',y',z)

$$B_{eff} = \left(B_0 - \frac{\omega}{\gamma} \right) \vec{k} + B_1 \vec{i}\,'$$



Precession of magnetisation (a) in the laboratory frame under the influence of longitudinal field B_0, and transverse field B_1, (b) in the rotating frame under the influence of field B_{eff} and (c) in the rotating frame when $B_0=\omega/\gamma$.

Blochsche Gleichungen

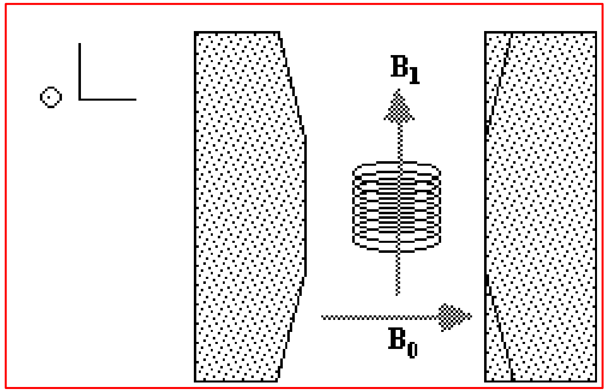
$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \qquad \frac{dM_y}{dt} = -\frac{M_y}{T_2} \qquad \frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1}$$

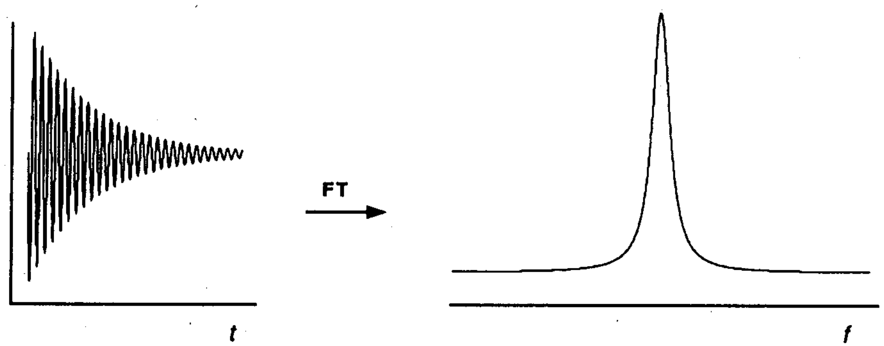
$$\frac{dM_x}{dt} = \gamma M_y (B_0 - \omega/\gamma) - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma M_z B_1 - M_y (B_0 - \omega/\gamma) - \frac{M_y}{T_2}$$

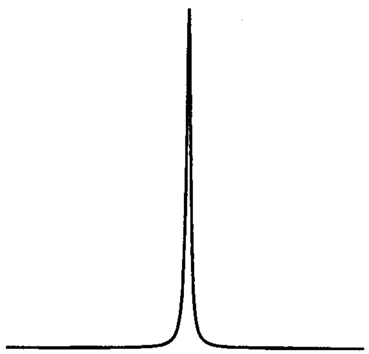
$$\frac{dM_z}{dt} = -\gamma M_y B_1 - \frac{(M_z - M_0)}{T_1}$$

NMR Relaxation

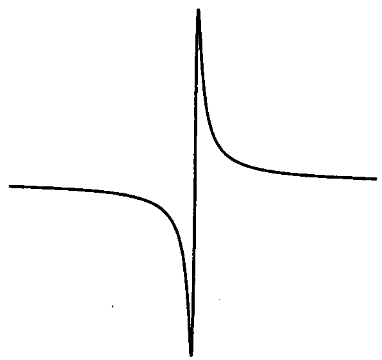




Freier Induktionsabfall

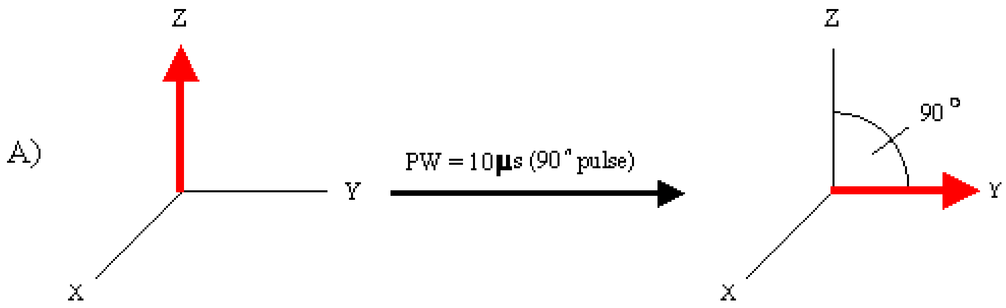


Absorptionssignal



Dispersionssignal

NMR-Pulse



$$\theta = \omega \; t_p = \gamma \, B_1 \, t_p$$

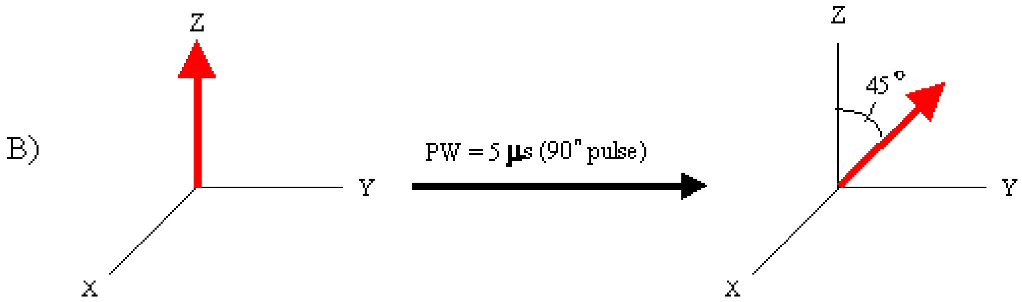
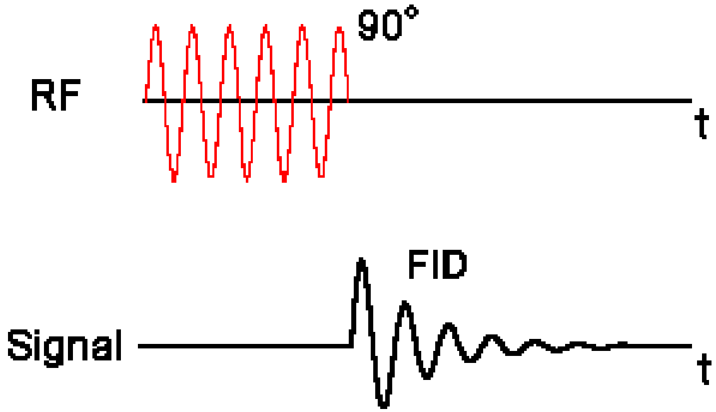
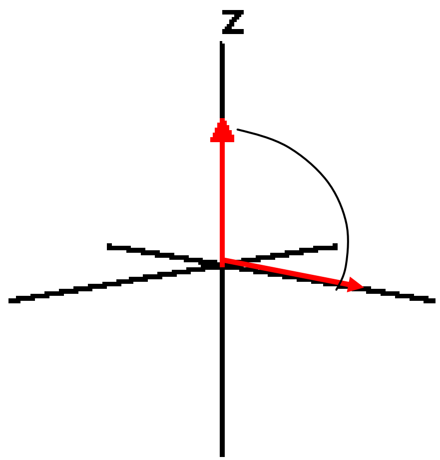


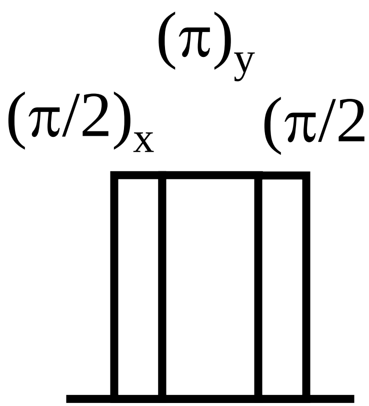
Figure 2. The effect of a pulse upon the nuclear spins. The average nuclear spin magnetization is represented by the heavy arrow.

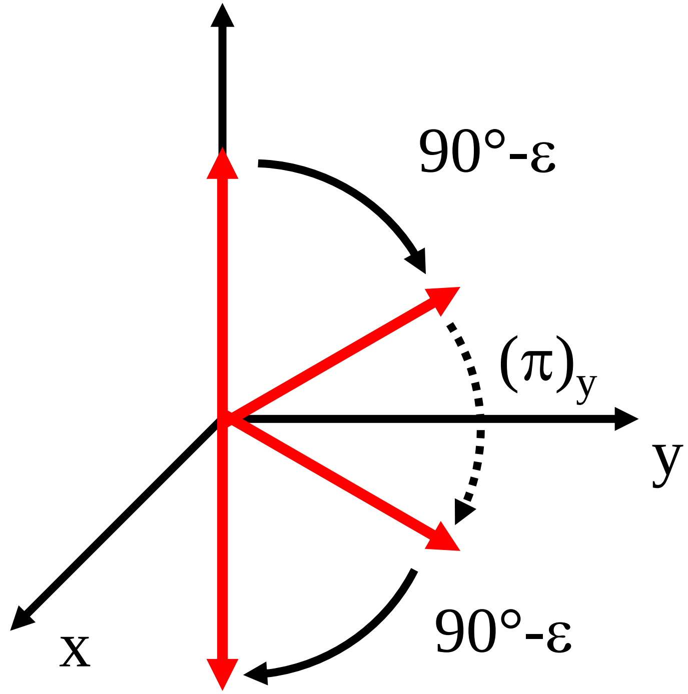
Relaxation: 90°-Puls





„Composite Pulses“

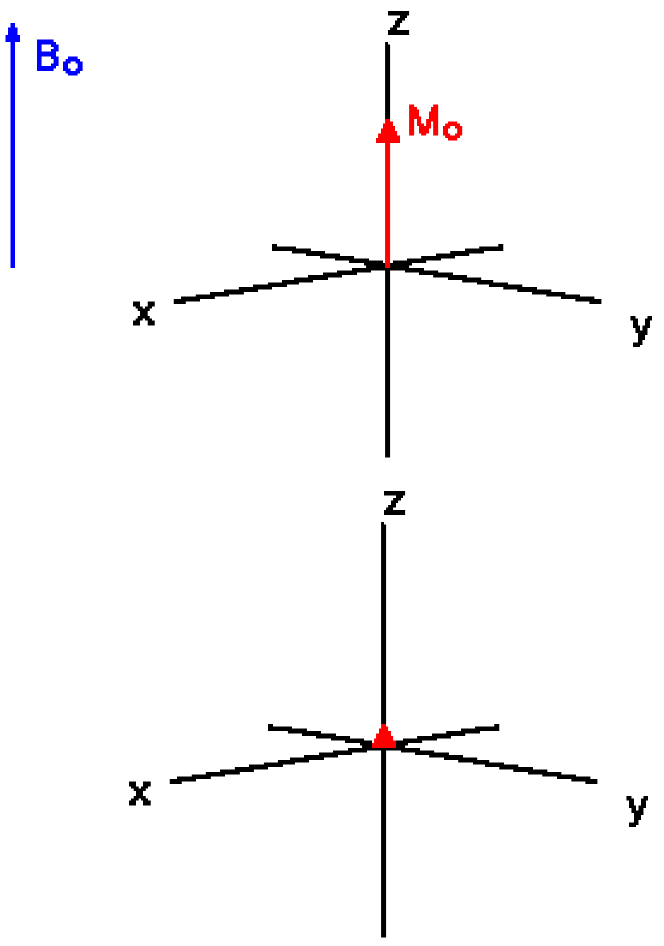


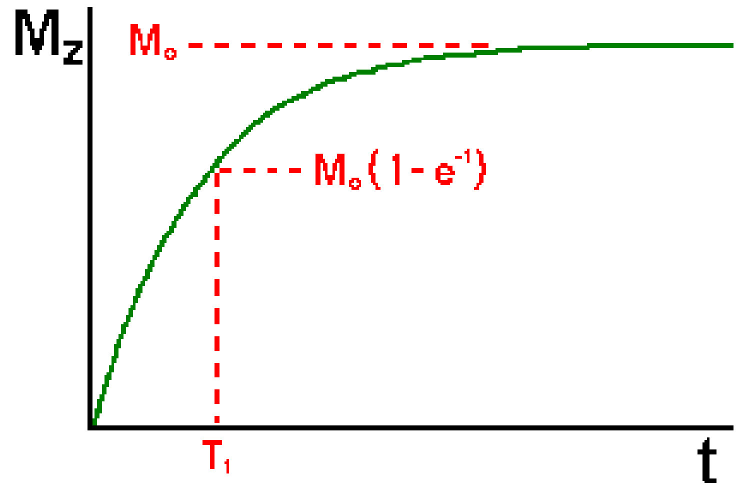


$(\pi/2)_x(4\pi/3)_y(\pi/2)_x$

Wartezeit: mindestens $5\times T_1$

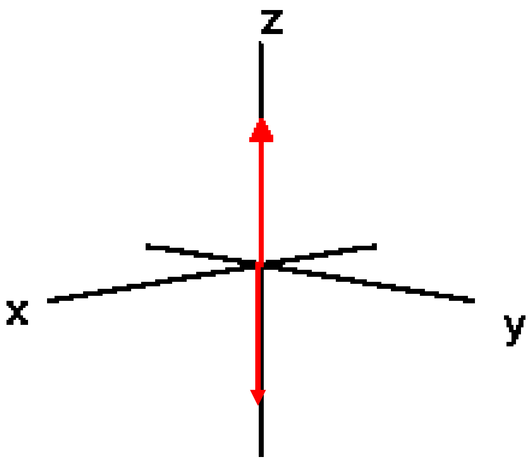
T_1-Relaxation (90°-τ-90°)

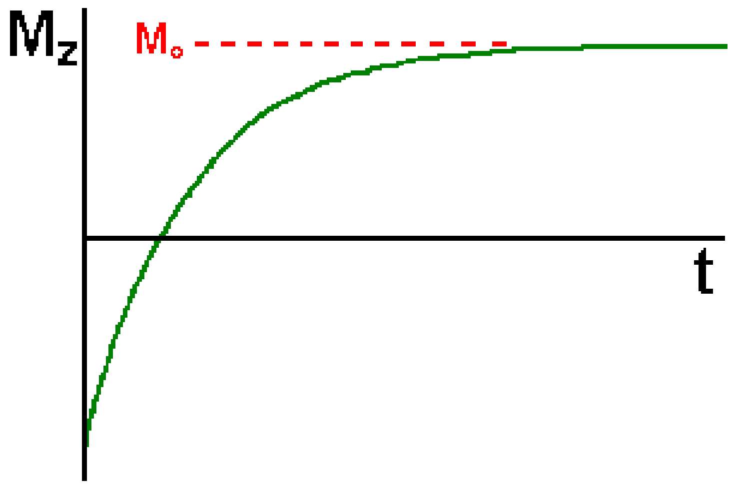




$$M_z = M_o\,(1 - e^{-t/T1})$$

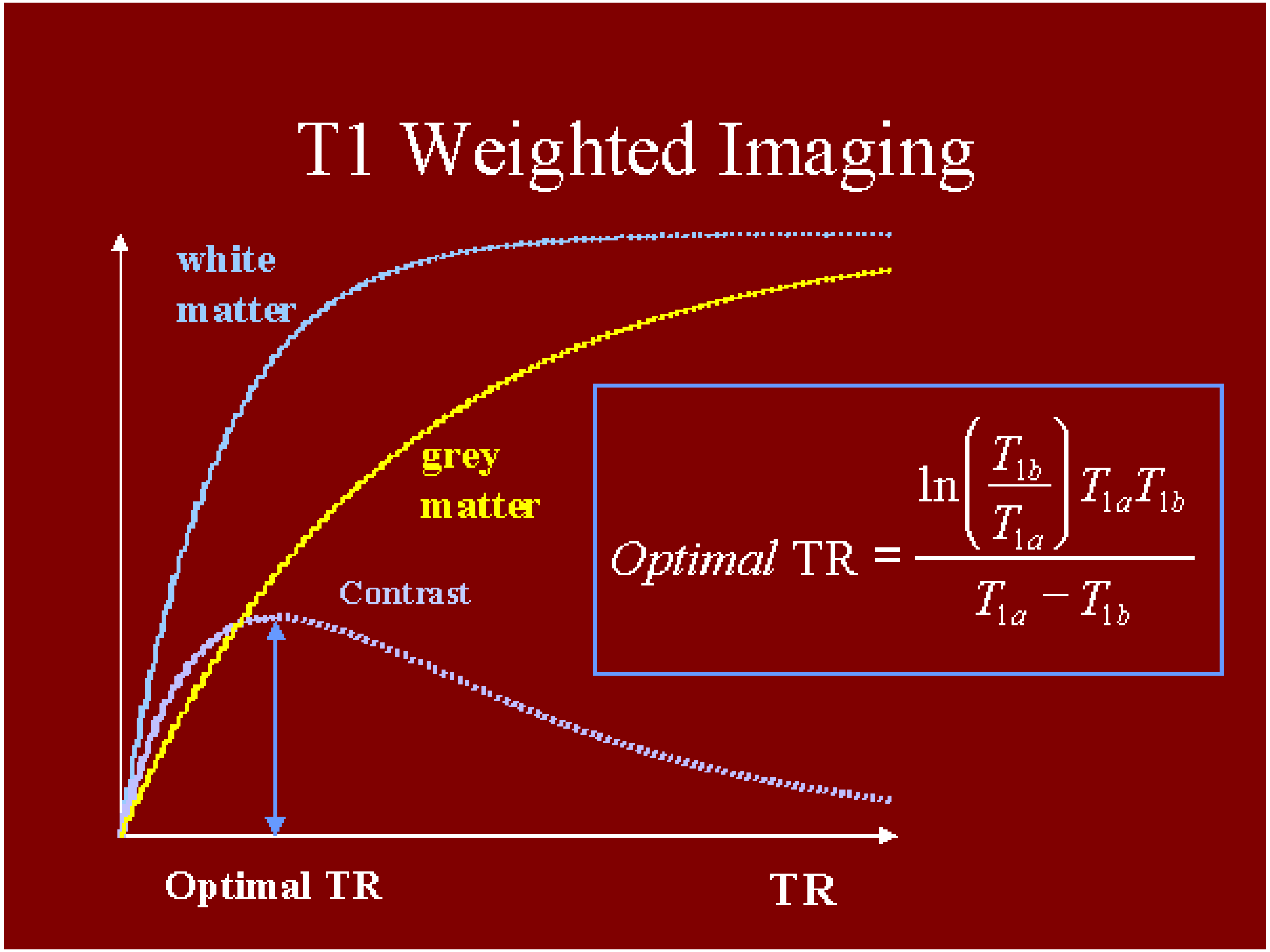
T_1-Relaxation (180°-τ-90°)





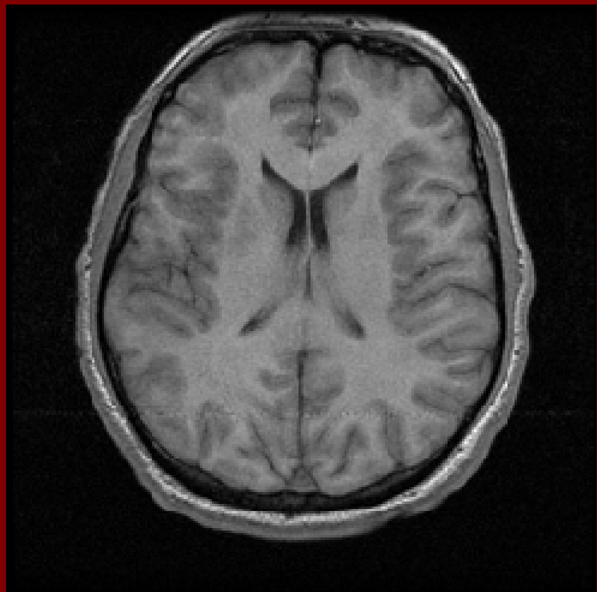
$$M_z = M_o (1 - 2e^{-t/T1})$$

T_1-Relaxation



T_1-Relaxation

T1 Weighted Image

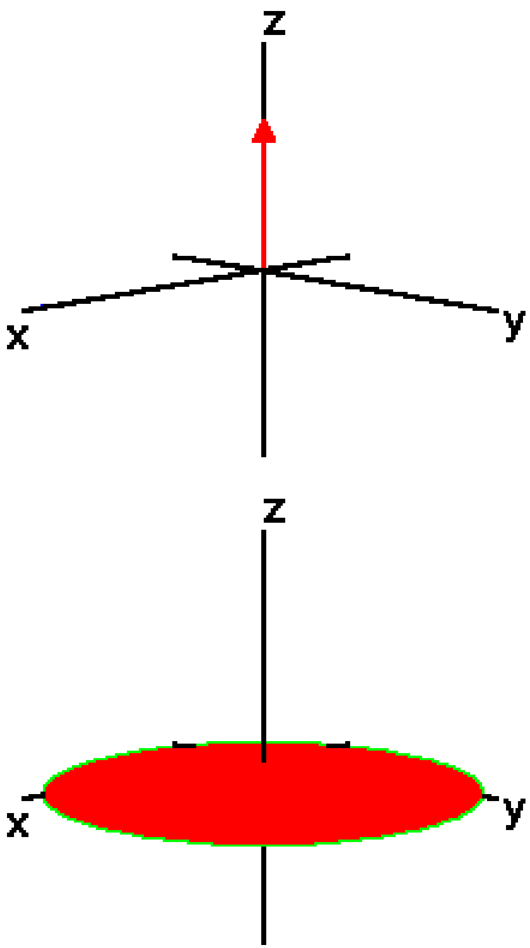


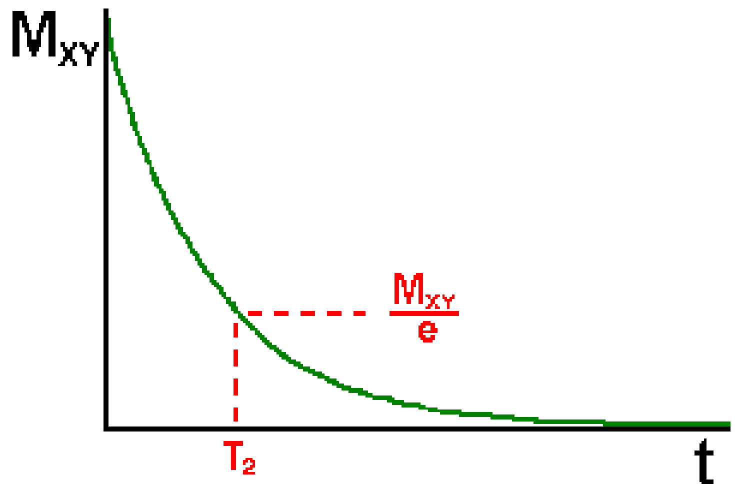
SPGR, TR=14ms, TE=5ms, flip=20°

	T_1/s	R_1/s^{-1}
white matter	0.7	1.43
grey matter	1	1
CSF	4	0.25

1.5T

T_2-Relaxation





$$M_{xy} = M_{xy}\, e^{-t/T2}$$

T_2-Relaxation

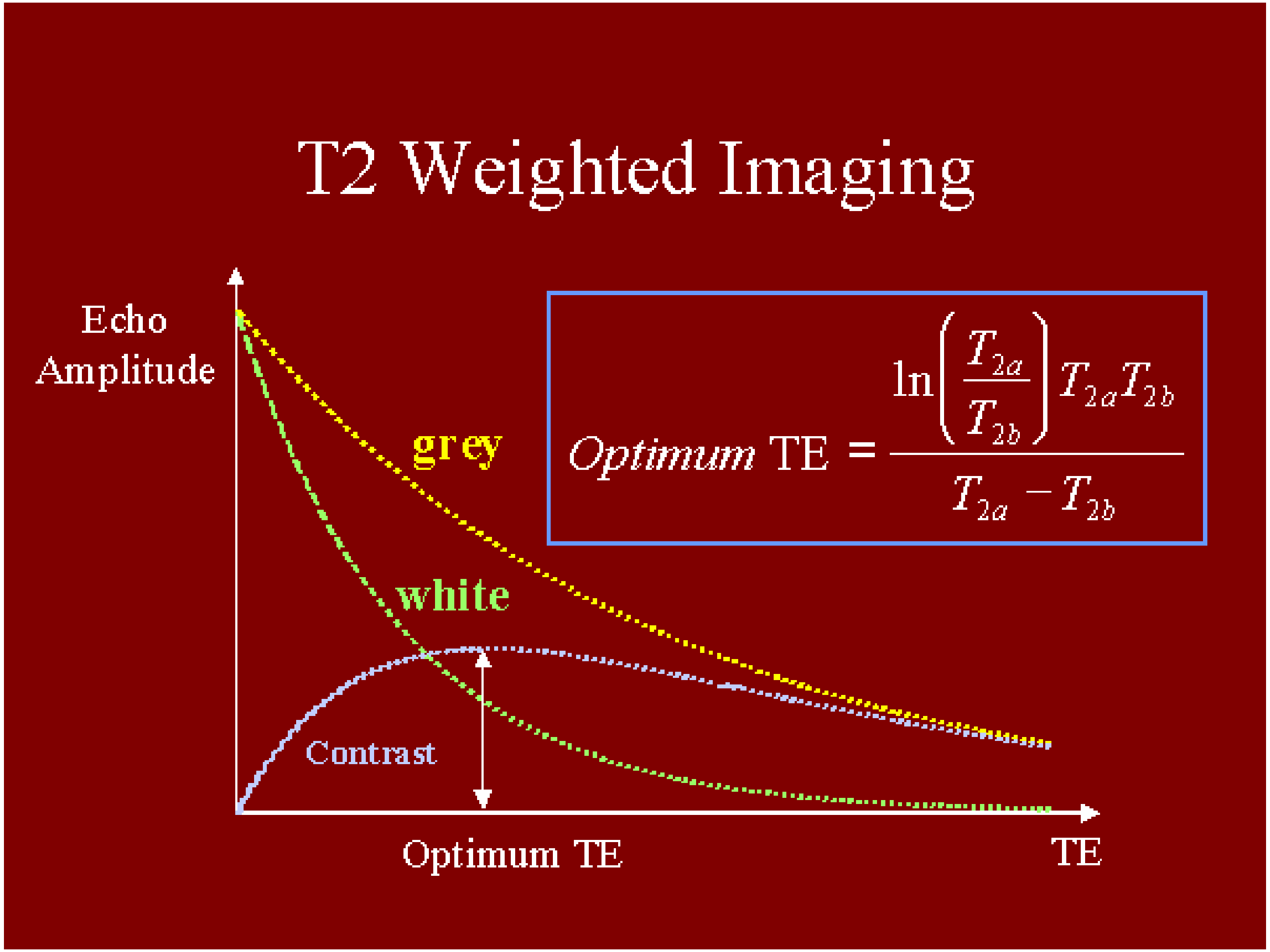
Zwei Faktoren tragen zum Zerfall der transversalen Magnetisierung bei:

1) Molekulare Wechselwirkungen (reiner T_2 Effekt)

2) Änderung in B_o (*inhomogener* T_2 Effekt)

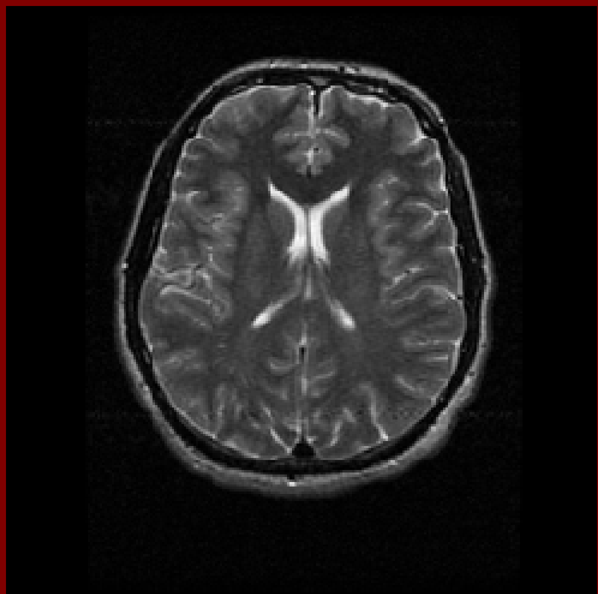
$$1/T_2^* = 1/T_2 + 1/T_2^{inhomo}$$

T_2-Relaxation



T_2-Relaxation

T2 Weighted Image

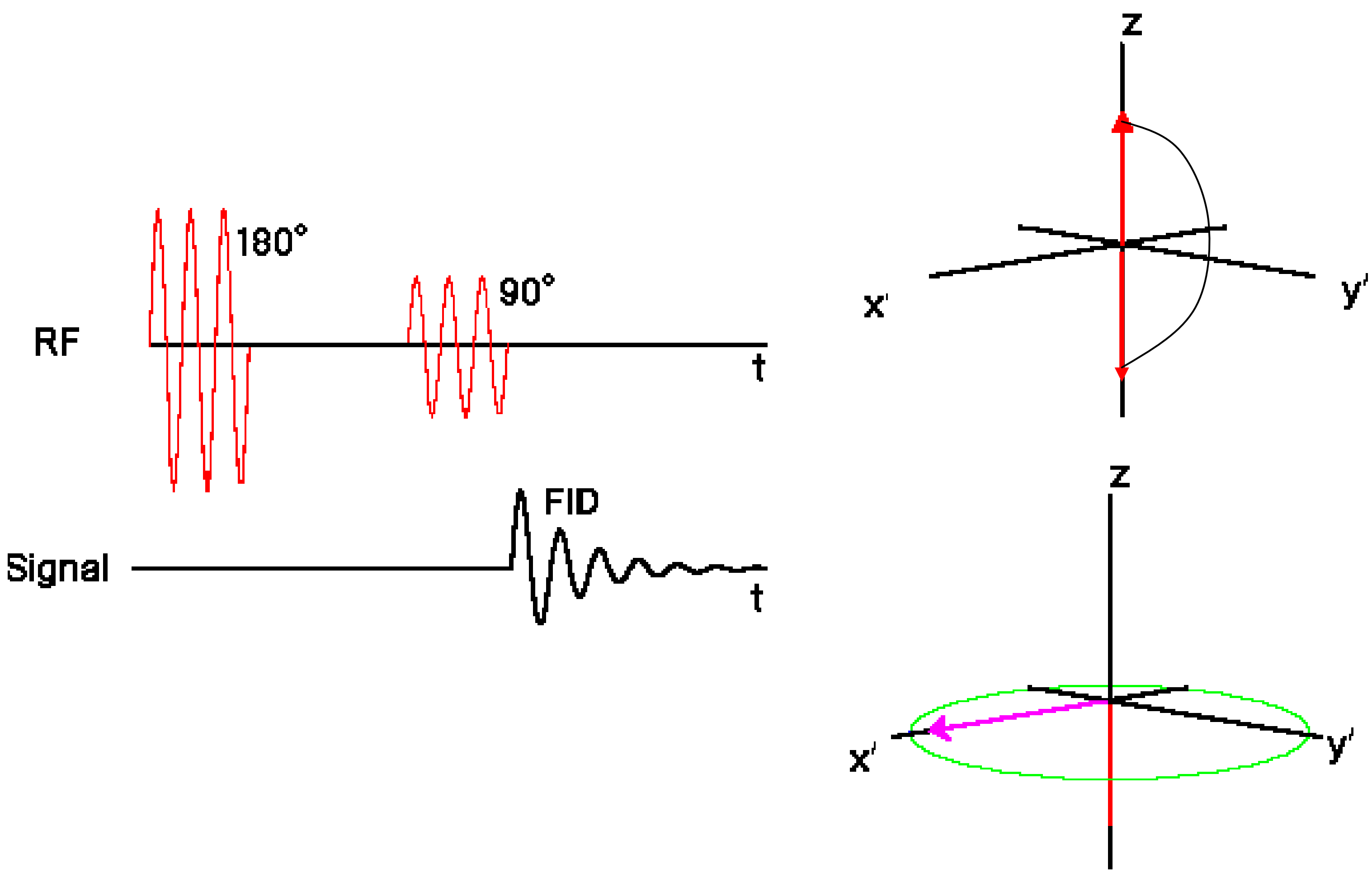


SE, TR=4000ms, TE=100ms

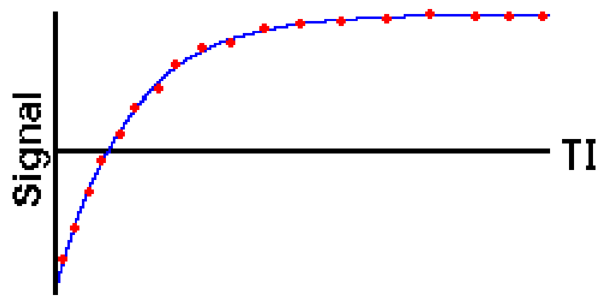
	T_2/ms
CSF	500
grey matter	80–90
white matter	70–80

1.5T

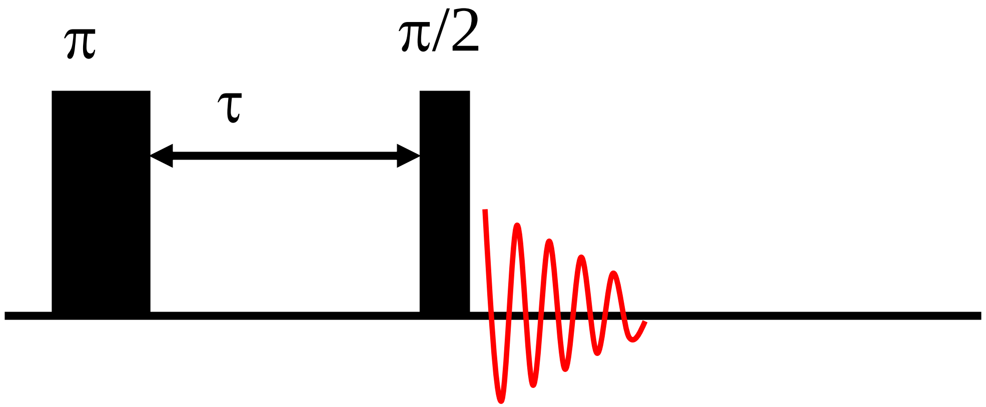
Inversion-Recovery 180°-τ-90°

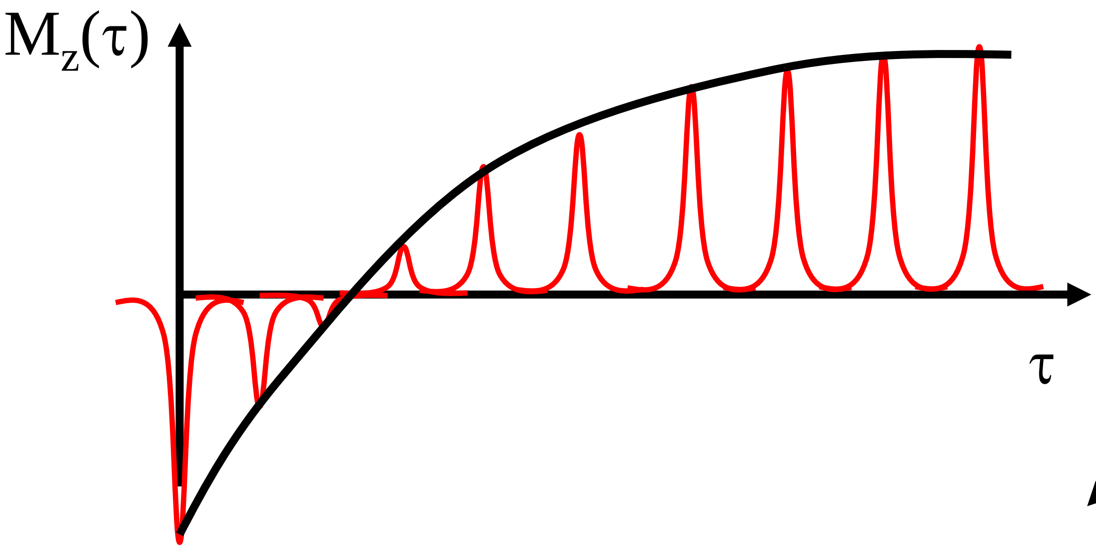


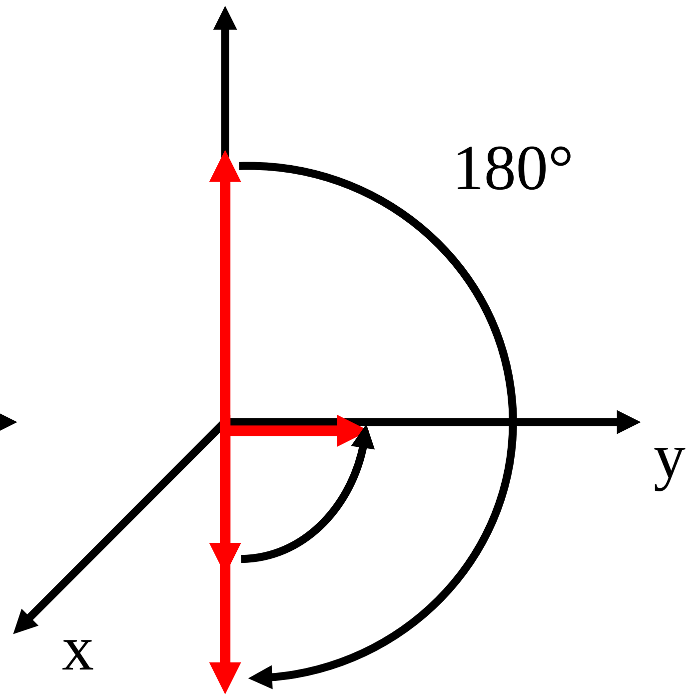
Inversion Recovery 180°-τ-90°



Inversion-Recovery-Methode

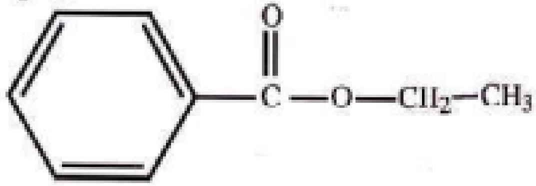


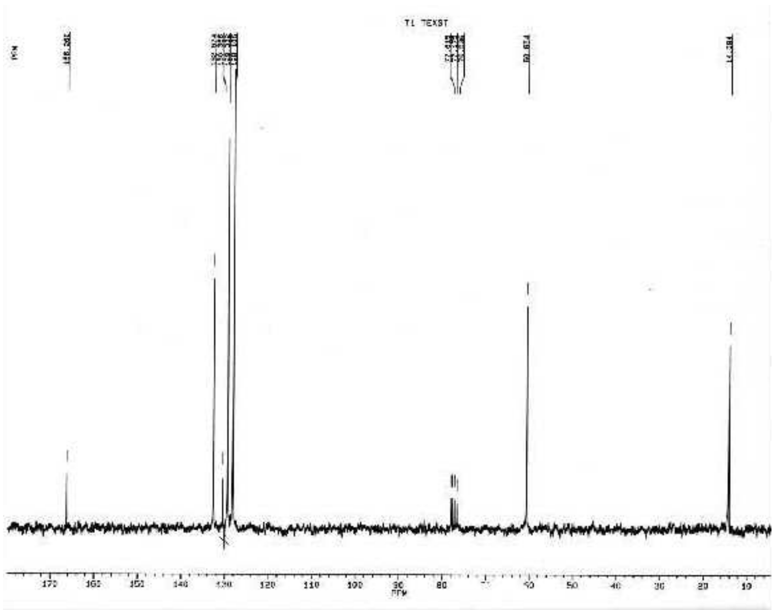


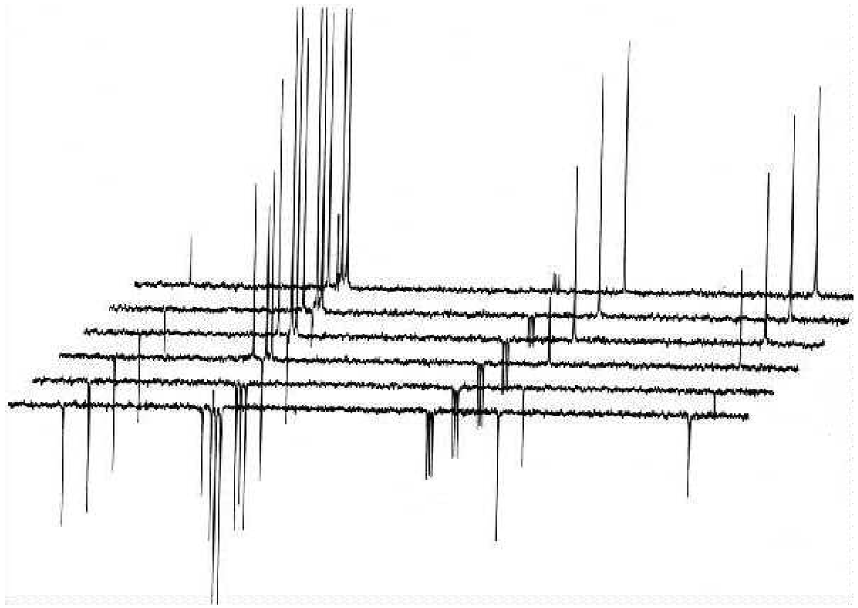


Inversion Recovery 180°-τ-90°

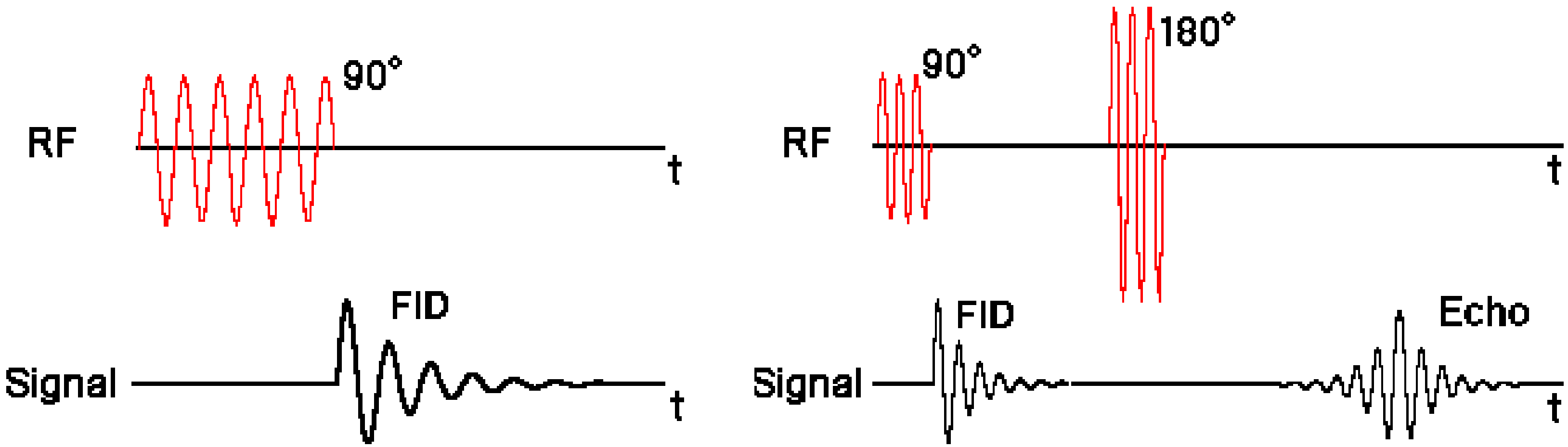
Ethyl Benzoate (T_1 sample):



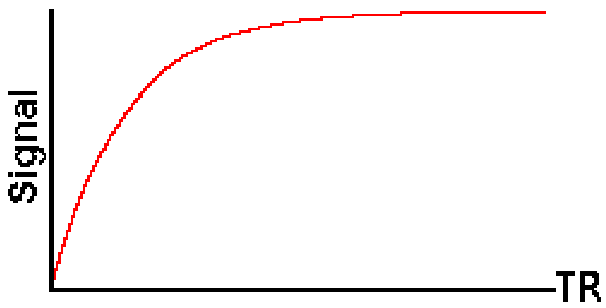


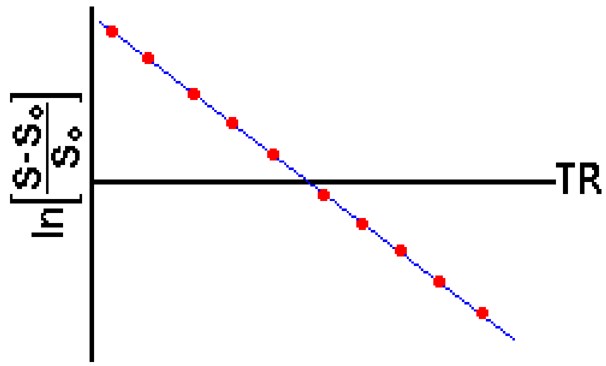


NMR T_2 und Refokussierung

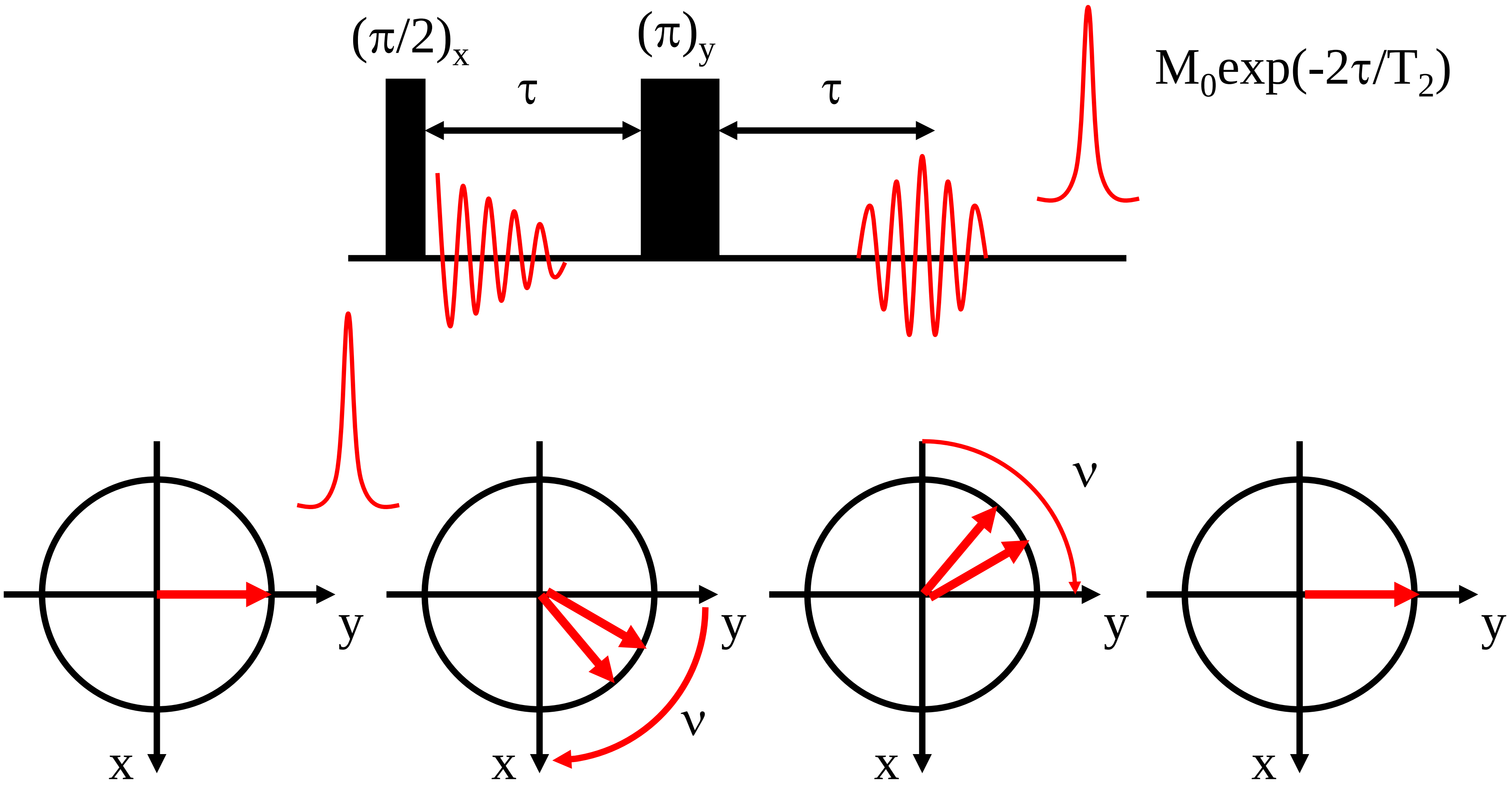


NMR T_2 und Refokussierung

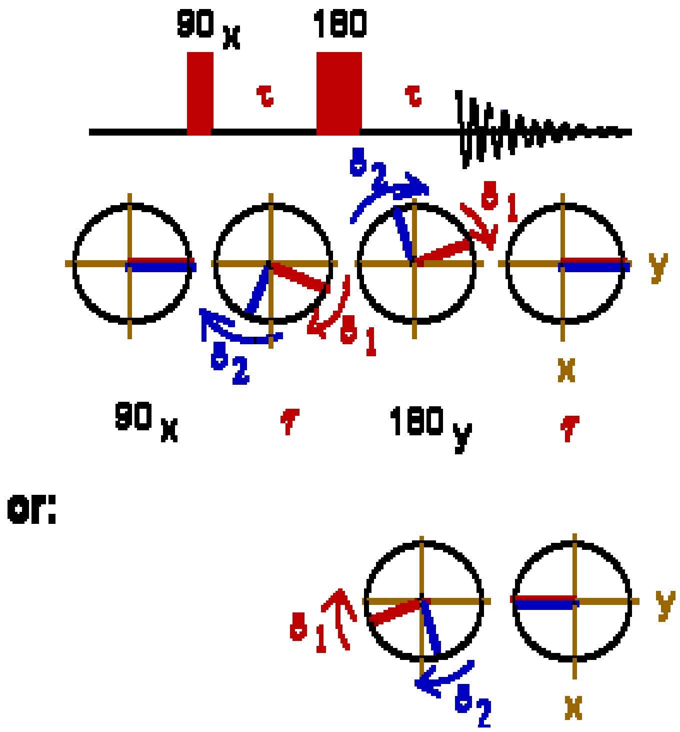




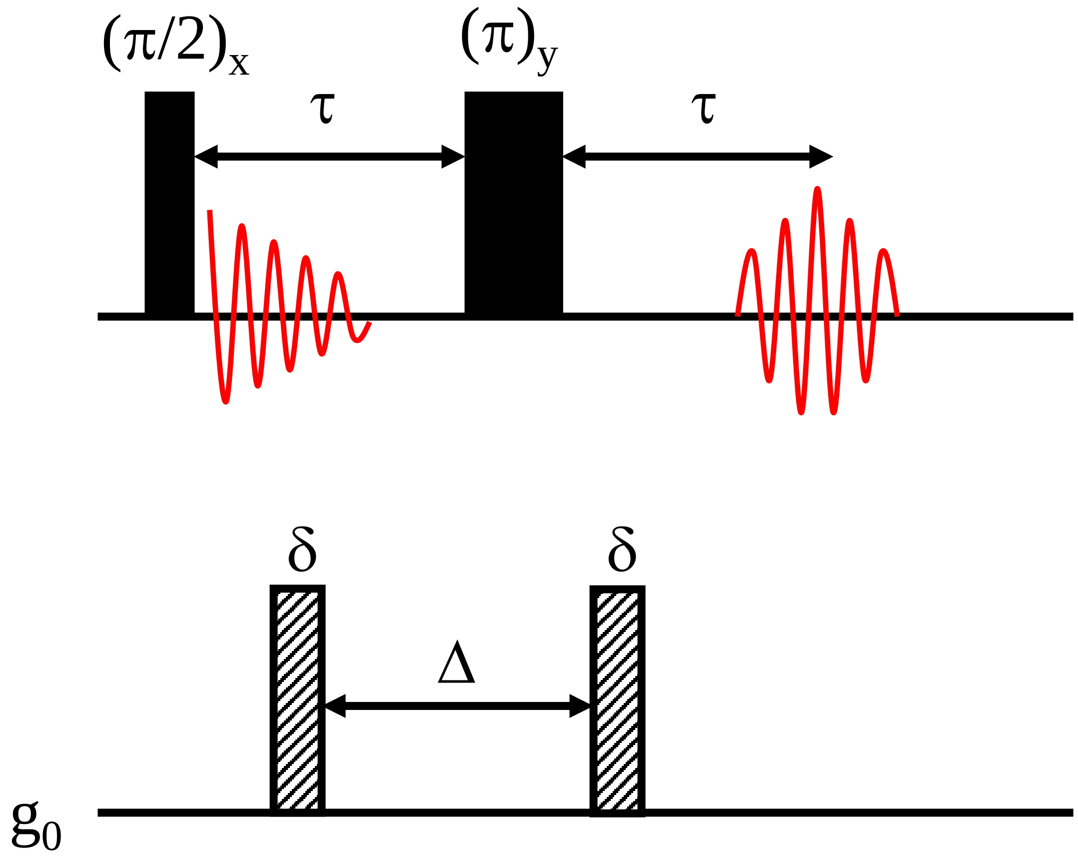
T_2 und Refokussierung



Hahnsches Spin-Echo



Linearer Gradient



Linearer Gradient

Linearer Gradientenpuls

Abstand zwischen Gradientenpulsen

$$M_t(2\tau) = M_0 \exp(-2\tau / T_2) \exp\left[-D\gamma^2 g_0^2 \delta^2 (\Delta + 2\delta / 3)\right]$$

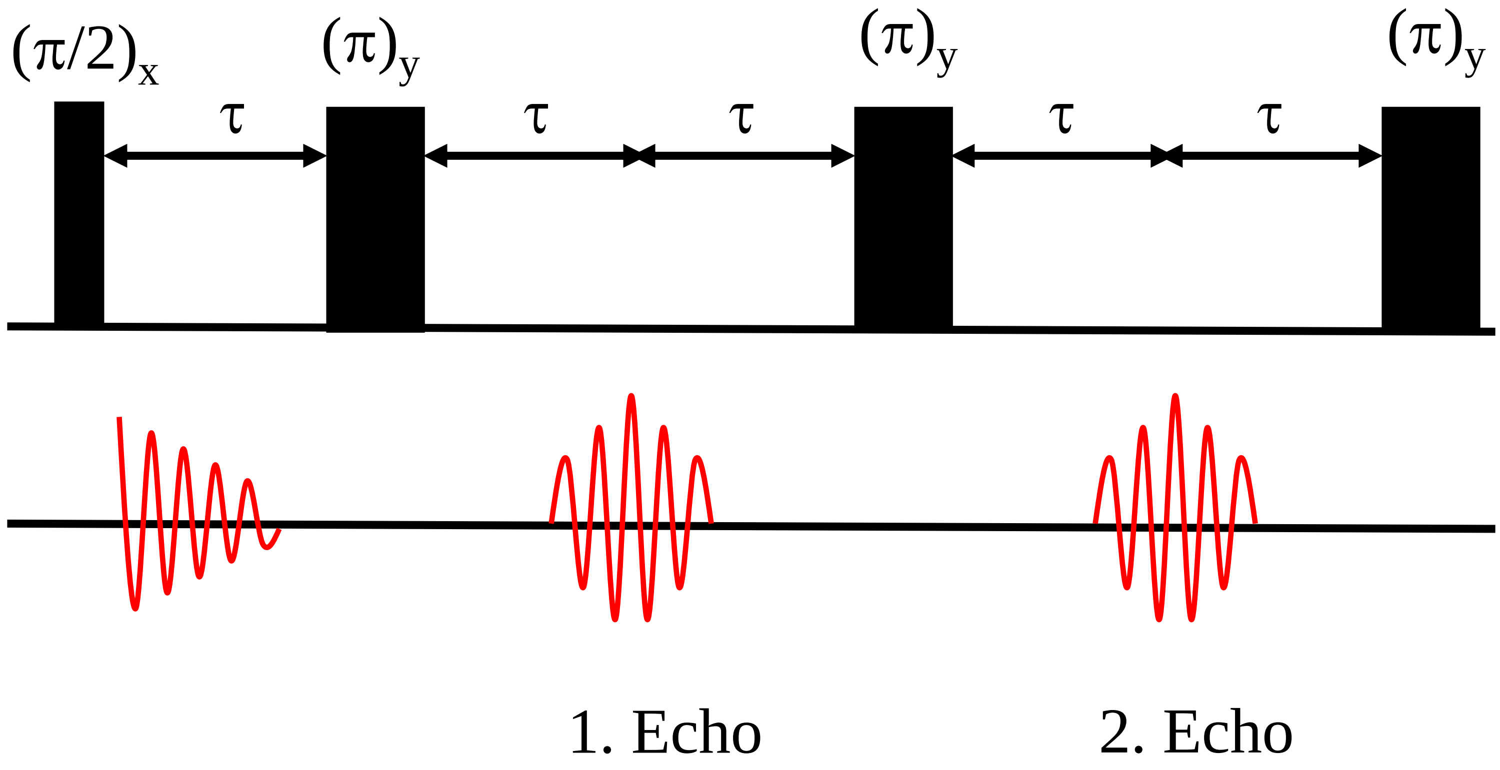
Gradientenpuls

$$M_t(\delta) \approx M_0 \exp(-2\tau / T_2) \exp\left[-D\gamma^2 g_0^2 \delta^2 \Delta\right]$$

Steigung: $-D\gamma^2 g_0^2 \Delta$

Selbstdiffusionskoeffizient D

Carr-Purcell-Meiboom-Gill Sequenz

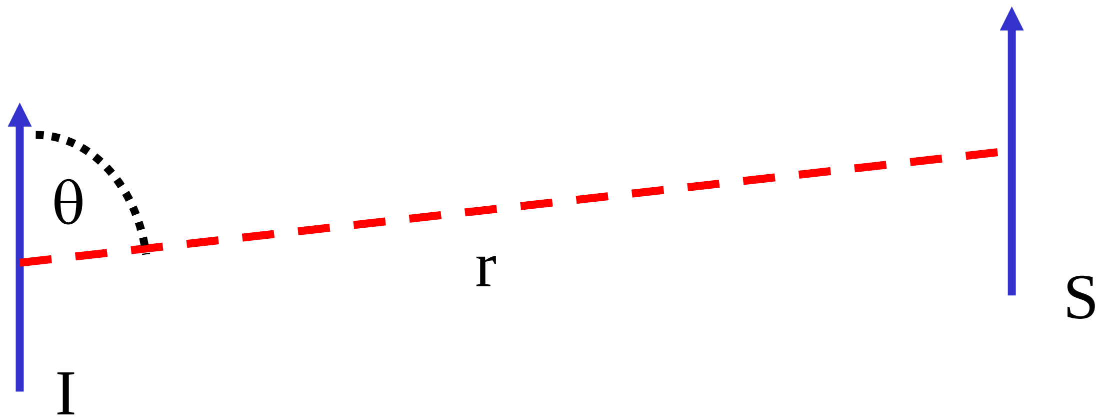


$$M_t(2n\tau) = M_0 \exp(-2n\tau / T_2)\exp\left[(-D\gamma^2 g_0^2 / 3)(2\delta\tau^3)\right]$$

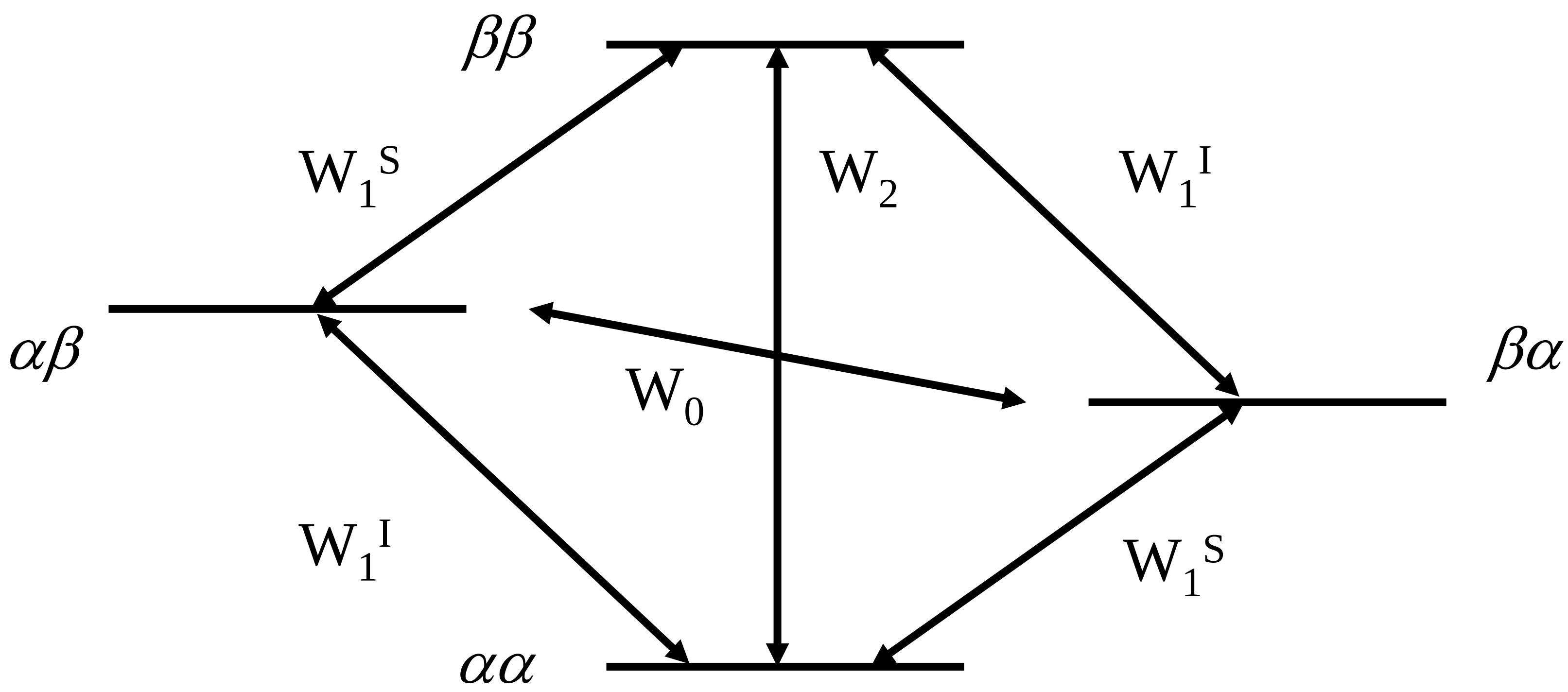
Relaxationsmechanismen

Spin-Gitter-Relaxationszeit T_1

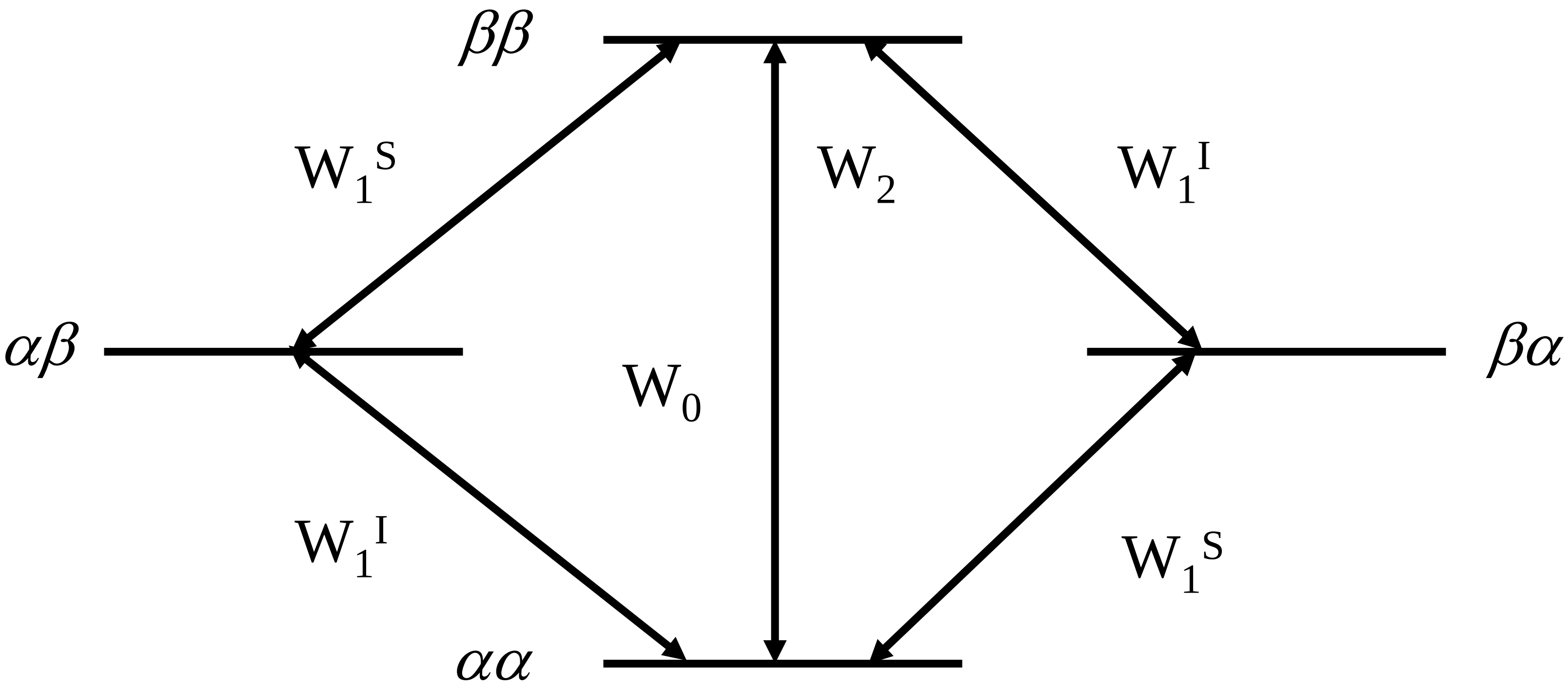
Spin-Spin-Relaxationszeit T_2



Relaxationsmechanismen



Relaxationsmechanismen



Spektraldichte

Spektraldichte: $$J(\omega) = \int_{-\infty}^{+\infty} K(\tau) e^{i\omega\tau} d\tau$$

Korrelationsfunktion: $$K_i(\tau) = \overline{Y_i(t) Y_i^*(t+\tau)}, \quad i = 0,1,2$$

$$Y_0 = r^{-3}(1 - 3\cos^2\theta)$$
$$Y_1 = r^{-3}\sin\theta\cos\theta\, e^{i\phi}$$
$$Y_2 = r^{-3}\sin^2\theta\, e^{2i\phi}$$

Spektraldichte

$$K_i(\tau) = K_i(0)\, e^{(-|\tau|/\tau_c)}$$

$$K_i(0) = \overline{Y_i(t)^2} = \overline{Y_i^{\,2}} = \int_0^{2\pi}\int_0^{\pi} Y_i^2 \sin\theta \, d\theta \, d\phi$$

$$K_0(0) = \frac{12}{15r^{-6}}, \quad K_1(0) = \frac{2}{15r^{-6}}, \quad K_2(0) = \frac{8}{15r^{-6}}$$

$$J(\omega) = \int_{-\infty}^{+\infty} K(\tau) e^{i\omega\tau} d\tau = \int_{-\infty}^{+\infty} K_i(0) e^{(-|\tau|/\tau_c)} e^{i\omega\tau} d\tau$$

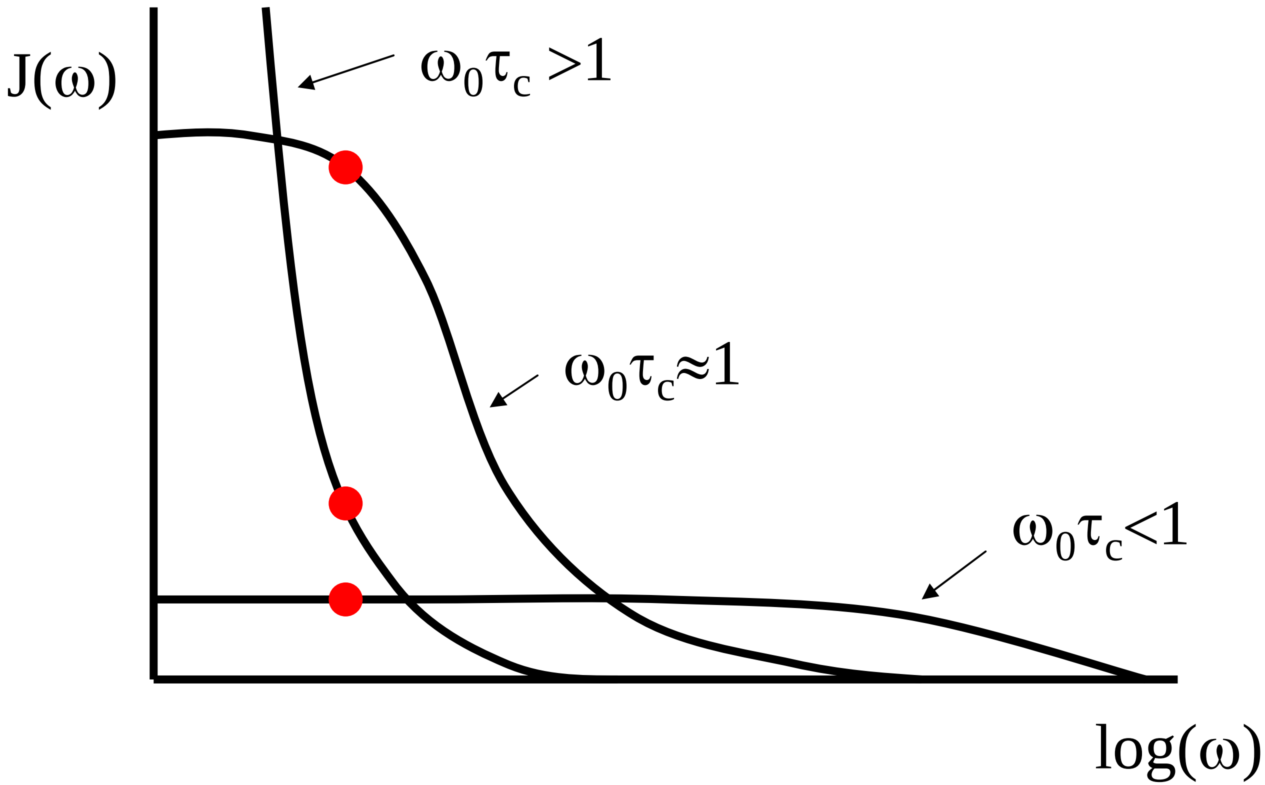
Spektraldichte

$$J_1 \propto \frac{3\tau_c}{r^6(1+\omega_I \tau_c{}^2)} \propto \frac{3\tau_c}{r^6}$$

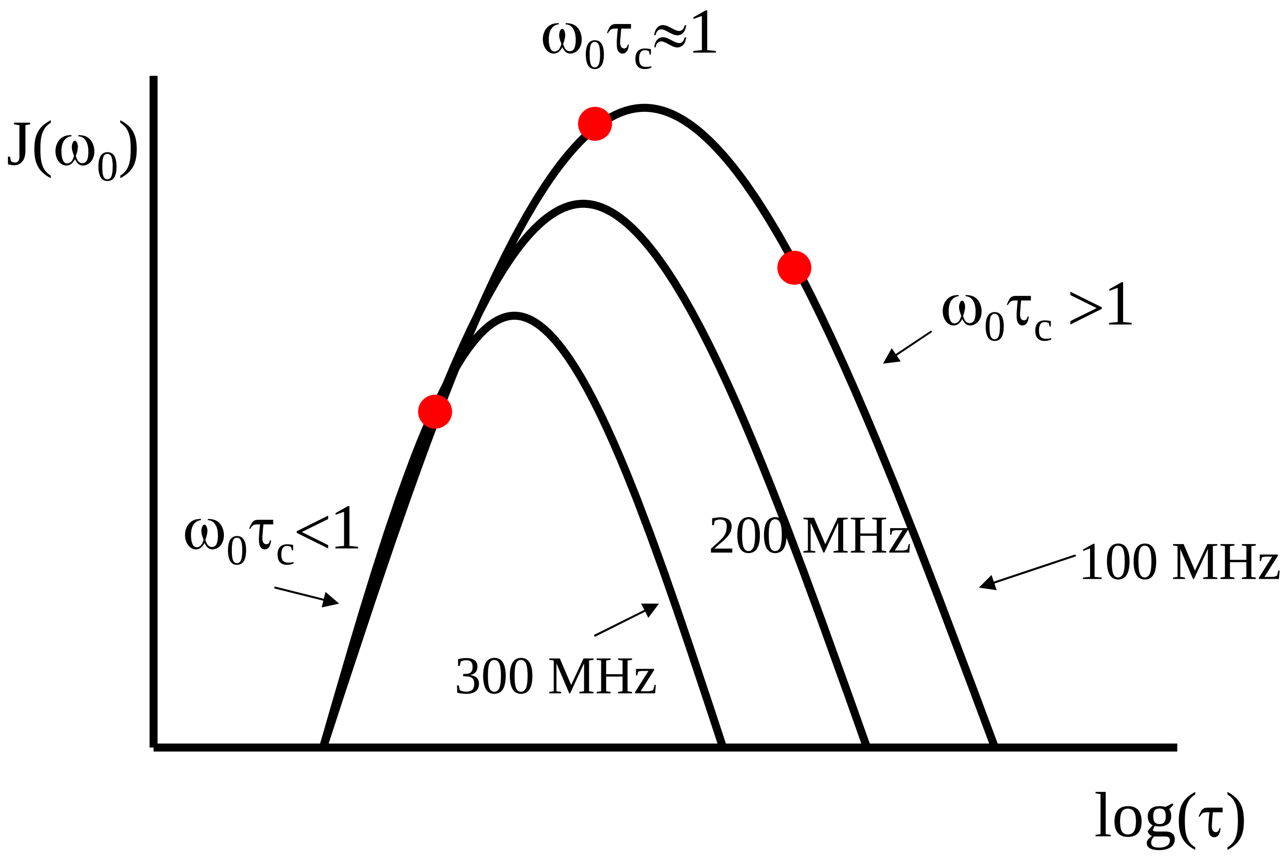
$$J_0 \propto \frac{2\tau_c}{r^6(1+(\omega_I - \omega_S)^2 \tau_c{}^2)} \propto \frac{2\tau_c}{r^6}$$

$$J_2 \propto \frac{12\tau_c}{r^6(1+(\omega_I + \omega_S)^2 \tau_c{}^2)} \propto \frac{12\tau_c}{r^6}$$

Spektraldichte



Spektraldichte



Spektraldichte

Spektraldichte:

$$J(\omega) = \int_{-\infty}^{+\infty} K(\tau) e^{i\omega\tau} d\tau$$

Korrelationsfunktion:

$$K_i(\tau) = \overline{Y_i(t) Y_i^*(t+\tau)}, \quad i = 0,1,2$$

$$K_0(0) = \frac{12}{15r^{-6}}, \quad K_1(0) = \frac{2}{15r^{-6}}, \quad K_2(0) = \frac{8}{15r^{-6}}$$

$$\boxed{J_i(\omega) = \left\langle \left| K_i(0) \right|^2 \right\rangle \cdot \frac{2\tau_c}{1 + (\omega\tau_c)^2}}$$

Spektraldichte