APPENDIX

During our ongoing research in stochastic modeling of publish/subscribe systems, we noticed (i) an error in the calculation of $p_s^f(k)$ for merging-based routing and (ii) that the calculation of $p_n^f(k)$ can be simplified for merging-based routing. Therefore, we now present substitutions for the two paragraphs in Subsects. 3.3.1 and 3.3.2 dealing with merging-based routing.

A. SECTION 3.3.1 – MERGING-BASED R.

In case of merging-based routing, a notification of class f with value x of the numeric attribute is propagated from broker B_i to B_j if in the subtopology $\mathcal{T}(B_i, B_j)$ there is (i) a subscription for the same filter class whose lower interval limit is smaller than or equal to x and (ii) a subscription for the same filter class whose upper interval limit is greater than or equal to x. Please note that these two conditions can also be satisfied by one single subscription. Let k be the number of subscriptions active for filter class f in the subtopology $\mathcal{T}(B_i, B_j)$. Then, the above two conditions can be rephrased as: a notification is *not* forwarded if either (i) all upper (and, thus, also lower) interval limits of all k subscriptions are smaller than x or (ii) all lower (and, thus, also upper) interval limits of all k subscriptions are larger than x. The probability for the former is x^{2k} and for the latter it is $(1-x)^{2k}$.

Therefore, the expected value of $p_n^f(k)$ is

$$p_n^f(k) = 1 - \int_0^1 x^{2k} + (1-x)^{2k} \mathrm{d}x = \frac{2k-1}{2k+1} \qquad (1)$$

B. SECTION 3.3.2 – MERGING-BASED R.

In case of merging-based routing, a newly issued or canceled subscription $[a, b]$ at broker B_i for filter class f causes a control message to be propagated to B_j if (i) a is smaller than the minimal lower limit of all subscriptions for filter class f active in the subtopology $\mathcal{T}(B_j, B_i)$ or (ii) b is greater than the maximal upper limit of these subscriptions. Please note that in case a subscription is canceled, all other instead of all subscriptions have to be considered. Let k be the number of subscriptions active for filter class f in the suptopology $\mathcal{T}(B_j, B_i)$ except the canceled subscription in case a subscription is revoked. Then, the above two conditions can be rephrased as: the subscription is propagated if (i) all lower (and, thus, also upper) interval limits of all k subscriptions are greater than a or (ii) all upper (and, thus, also lower) interval limits of all k subscriptions are smaller than b. The probability for the former equals $(1-a)^{2k}$ and the probability for the latter is b^{2k}. However, from these probabilities we have to subtract the probability that both conditions are satisfied. In this case, all interval limits of the k subscriptions fall inside the interval $[a, b]$. The probability for this case is $(b-a)^{2k}$.

Therefore, the expected value of $p_s^f(k)$ is

$$\begin{aligned} p_s^f(k) &= 2 \cdot \int_{a=0}^1 \int_{b=a}^1 (1-a)^{2k} + b^{2k} - (b-a)^{2k} \,\mathrm{d}b\,\mathrm{d}a \\ &= \frac{1+4k}{1+3k+2k^2} \end{aligned} \qquad (2)$$

The new formula for $p_s^f(k)$ increases the forwarding probability of subscriptions and unsubscriptions for merging-based routing. However, the introduced error is only relevant for small values of k and, thus, does not influence the contribution of the paper.