HDLS: Improved Localization via Algorithm Fusion

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***Abstract*—Wireless Sensor Networks (WSNs) have been of high interest during the past couple of years. One of the most challenging tasks of WSN research is still location estimation. As a well performing fine grained localization approach, Distributed Least Squares (DLS) was introduced, splitting the costly localization process in a complex *precalculation* and a simple *postcalculation*, which is performed on constrained sensor nodes. Nevertheless, as size of precalculation and consequently, cost of computation and communication are growing with network size, it was shown that this algorithm is unsuitable for large WSNs. This restriction has been overcome by scalable DLS (sDLS), which enables to use the idea of DLS in large WSNs for the first time. Although cost of computation of sDLS is independent of the network size, it was relatively high, due to costly matrix updates. Consequently, this cost was reduced by sDLS with normal equation (sDLS^{ne}), circumventing the updates. Unfortunately, sDLS^{ne} comes along with a decreased localization accuracy. The approach, presented in this work, combines the efficient sDLS^{ne} approach with various coarse grained localization techniques to improve localization accuracy. The resulting localization accuracy overcomes the efficient sDLS^{ne} approach as well as the more precise sDLS approach, while cost of computation still outperforms sDLS.**

***Keywords*-wireless sensor networks, localization, scalability.**

I. Introduction

Recent technological advances enabled development of tiny wireless devices, which are able to sense their environment, compute simple tasks and exchange data among each other. Interconnected assemblies of such devices, called Wireless Sensor Networks (WSNs), are commonly used to observe large inaccessible areas. In many applications of WSN, knowledge of nodes' locations is mandatory for a meaningful interpretation of measured data. In addition, location-awareness is also necessary for geographic routing [1][2] or location based clustering [3]. Due to existing limitations in terms of size, financial cost and energy consumption, local positioning within the network is preferred over utilizing Global Navigation Satellite Systems (GNSSs) like GPS [4]. Therefore, the presence of location-aware sensor nodes, referred to as beacon nodes, is typically assumed. The remaining nodes, which we refer to as blind nodes, are assumed to use communication and any kind of distance estimation or neighborhood information to estimate their positions with the help of beacon nodes.

Localization algorithms can be divided into centralized and decentralized on the one hand, and fine-grained and coarse-grained on the other hand. Coarse-grained approaches like Centroid Localization (CL) [5], Weighted Centroid Localization (WCL) [6] and Adaptive Weighted Centroid Localization (AWCL) [7] often abstain from exact distances, require less communication and computation, and provide lower precision estimates. In contrast, fine-grained approaches use costly computations and distance estimations to achieve localization with high precision. High precision and low complexity have been firstly combined by Distributed Least Squares (DLS) [8], which splits the costly localization calculation into *precalculation* and *postcalculation*. Independent from a specific blind node, the complex precalculation is performed on a high performance sink. The remaining postcalculation is less complex and performed on resource-constrained blind nodes.

The concept of DLS has been adapted by scalable DLS (sDLS) [9], which enables the idea of DLS to be used in large WSNs. In contrast to DLS, sDLS provides costs of computation and communication, incurred on blind nodes, which are independent from network size, i.e., independent from total number of beacon nodes. This is achieved by use of individual precalculations instead of one global precalculation. A fundamental enhancement is given by sDLS with normal equation (sDLS^{ne}) [10], which significantly reduces the cost of computation by circumventing costly updates, introduced with sDLS.

Using sDLS, blind nodes are assumed to choose one precalculation out of several precalculations provided by neighbouring beacon nodes, according to their distances. Commonly, the set of beacon nodes included in the chosen precalculation differs from the set of beacon nodes within a blind node's communication range. This causes a suboptimal localization accuracy and offers possibilities for further improvements. The present work combines multiple position estimates, based on sDLS^{ne}, by use of coarse grained localization techniques, to improve localization accuracy.

The remainder of the paper is organized as follows. Section II covers basic informations about sDLS algorithms. In Section III, the new hybrid localization approach is presented in various variants, using several optimizing parameters. Section IV covers performed simulations. Simulation results are presented in Section V. Finally, the presented work is summarized in Section VI.

II. Related Work

The DLS algorithm was developed to diminish trade off between precision and cost of localization [8]. It provides localization with high precision at low cost. The basic idea of splitting the calculation into precalculation and postcalculation was adapted by sDLS and its successor sDLS^{ne} to support large WSNs with network size independent cost for blind nodes. Both approaches are briefly described in this section.

The system of equations, which have to be solved for localization of a blind node is originally built by distance equations as given in equation (1).

$$(x-x_i)^2+(y-y_i)^2 = r_i^2 \quad (i \in I; I = \{1,2,\ldots,m\}) \tag{1}$$

Here x and y give the unknown position of a blind node. The known position of a beacon node is denoted as x_i and y_i, while the distance between both nodes is denoted as r_i. The number of beacon nodes utilizable for localization is given as m.

This system of equations is linearized by use of a linearization tool [11], using one beacon node as linearizer, denoted with index L. After restructuring, the system of equations consists of equations as given in equation (2), where r_L denotes the distance between blind node and linearizer, r_i is the distance between blind node and beacon node, and d_{iL} denotes the distance between linearizer and beacon node.

$$\begin{aligned} b_{iL} &= (x-x_L)(x_i-x_L)+(y-y_L)(y_i-y_L) \\ &= \frac{1}{2}\left[r_L^2 - r_i^2 + d_{iL}^2\right] \end{aligned} \tag{2}$$

After further restructions, the system of equations matches the matrix form $\mathbf{A}\mathbf{x} = \mathbf{b}$, using $\mathbf{A}$, $\mathbf{x}$ and $\mathbf{B}$ as given in equation (3).

$$\begin{gathered} \mathbf{A} = \begin{pmatrix} x_{k_1}-x_L & y_{k_1}-y_L \\ x_{k_2}-x_L & y_{k_2}-y_L \\ \vdots & \vdots \\ x_{k_n}-x_L & y_{k_n}-y_L \end{pmatrix}, \\ \mathbf{x} = \begin{pmatrix} x-x_L \\ y-y_L \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_{k_1L} \\ b_{k_2L} \\ \vdots \\ b_{k_nL} \end{pmatrix} \end{gathered} \tag{3}$$

Here, the beacon nodes, used for localization, are denoted with indices $K = \{k_1, k_2, \ldots, k_n\}$ with $K = \{I \setminus L\}$. Matrix $\mathbf{A}$ of equation (3) only consists of beacon position data, while $\mathbf{b}$ contains distances between beacon nodes and blind nodes. Therefore, calculations on matrix $\mathbf{A}$ are to be performed as part of the precalculation at a powerful sink outside the WSN. The localization will be finalized on each blind node by performing the remaining part of the calculation.

To solve the linear system of equations, using normal equations, equations (4a) to (4c) are used. While (4a) shows the entire equation, (4b) presents the precalculation, performed on the sink, and (4c) presents the postcalculation, performed on blind nodes.

$$\mathbf{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\frac{1}{2}\left[r_L^2 - \mathbf{r}^2 + \mathbf{d}^2\right] \tag{4a}$$

$$\begin{aligned} \mathbf{A}_p &= \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T \\ \mathbf{d}_p &= \mathbf{d}^2 \end{aligned} \tag{4b}$$

$$\mathbf{x} = \mathbf{A}_p\frac{1}{2}\left[r_L^2 - \mathbf{r}^2 + \mathbf{d}_p\right] \tag{4c}$$

The main difference between DLS and sDLS^{ne} is given by number and size of precalcultions. Regarding beacon nodes in a WSN, $\mathbb{G}$ is considered as the global set of all beacon nodes and $\mathbb{L}_i \subseteq \mathbb{G}$ denotes a local set of beacon nodes within the communication range of beacon node i. While DLS uses only one precalculation, including all beacon nodes, i.e., equation (3) with conditions $K = \{\mathbb{G} \setminus L\}$ and $L = 1$, sDLS^{ne} uses individual precalculations for all beacon nodes, i.e., $|\mathbb{G}|$ precalculations using $K = \{\mathbb{L}_i \setminus L\}$, $L = i$, $\forall i \in \mathbb{G}$. Therefore, the sDLS^{ne} algorithm starts with an additional discovery phase to find other beacon nodes in one hop distance, as illustrated in Figure 1.

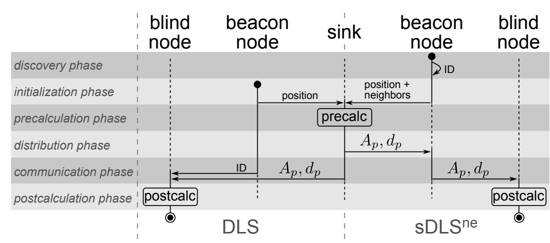

Figure 1. Algorithmic comparison of DLS and sDLS^{ne}

Furthermore, DLS needs an explicit communication with all beacon nodes during the communication phase for distance estimation. Using sDLS^{ne}, this is an implicit process as each blind node receives precalculations from beacon nodes in its own communication range.

Using sDLS^{ne}, each beacon node provides its own precalculation, which would perfectly fit for a blind node on the same position. From all offered precalculations, blind nodes are expected to chose the one of the closest beacon node.

III. Hybrid Localization Approach

The original intention of sDLS was to use exactly those beacon nodes, which are located within the communication range of the blind node, attempting to estimate its own position. To achieve this goal, a blind node is expected to choose the precalculation provided by the beacon node closest to its own position. Consequently this precalculation includes most of the beacon nodes within the blind node's communication range. Nevertheless, in most cases some beacon nodes included in this precalculation are outside the communication range of the blind node and vice versa. While sDLS locally updates this precalculation, by use of matrix updates, to achieve the initial intention, sDLS^{ne} estimates the unknown position with this unprecise precalculation.

Due to this displaced set of beacon nodes as well as the high influence of the node geometry, especially the given choice of the linearizing beacon node, the resulting position estimation tends to be drawn in the direction of this beacon node. In addition, the used distance estimation also causes an impairement of the position estimation. Furthermore, a defective distance estimation may cause the blind node to spuriously choose a precalculation of a beacon node, which is not the closest.

The aim of Hybrid Distributed Least Squares (HDLS) is to use multiple precalculations of nearby beacon nodes. The resulting position estimates, according to each chosen precalculation, serve as tentative results. These results can be seen as virtual beacon nodes. They will be combined to a final position estimate using coarse grained localization techniques. For that aim, various approaches have been studied in this work. The used coarse grained localization approach presents only one factor, that influences the resulting accuracy. The following factors, studied in our work, are to be further explained in this section:

Strategy:	number of virtual beacon nodes
Technique:	used coarse grained approach
Weightage:	used weight factor
Reduction:	reduction part, used by AWCL
Approximation:	distance approximation of inaccessible beacon nodes

A. Virtual Beacon Strategy

To control the number of virtual beacon nodes that are to be created using sDLS^{ne}, the following strategies have been investigated:

Closest Two – Virtual beacon nodes are created from precalculations of the two closest beacon nodes.
Closest Three – Virtual beacon nodes are created from precalculations of the three closest beacon nodes.
Great Deal – Virtual beacon nodes are to be created, using precalculations of all beacon nodes in range.
Range Based – Beacon nodes in a range, given as a multiple of the distance to the closest beacon node, are used for creation of virtual beacon nodes. This strategy extends the before mentioned strategies, which serve as upper bound. Within our investigations, this range has been varied from 125% up to 250% of the closest beacon node.

B. Coarse Grained Estimation Technique

Created virtual beacon nodes are combined to a resulting position estimation P_b using coarse grained localization techniques. The following techniques have been studied:

CL – The plain Centroid Localization (CL) approach is used to combine the virtual beacon nodes, i.e., unweighted arithmetic mean is used as given in equation (5). Here, $\mathbb{V}$ indicates a set of given virtual beacon nodes and P indicates a position.

$$P_b = \frac{1}{|\mathbb{V}|}\sum_{i\in\mathbb{V}} P_i \tag{5}$$

WCL – Virtual beacon nodes are combined using Weighted Centroid Localization (WCL) as given in equation (6). Suitable substitutions for weight w_i are to be presented subsequently. Common weights rely on measured distances or received signal strength (RSS).

$$P_b = \frac{\sum_{i\in\mathbb{V}} P_i * w_i}{\sum_{i\in\mathbb{V}} w_i} \tag{6}$$

AWCL – Virtual beacon nodes are combined by use of Adaptive Weighted Centroid Localization (AWCL). While WCL simply gives more influence to closer beacon nodes, i.e., beacon nodes with higher weight, the idea of AWCL is to give more influence to the difference of given weights. Therefore, if the weights, e.g., RSS, of beacon nodes in range are similar to each other, they are to be reduced by a reduction part q of the smallest weight, with $\{q \in \mathbb{R} | 0 \leq q \leq 1\}$, as illustrated in Figure 2. Otherwise, i.e., in case of high differences within the weights, AWCL inherently acts as WCL.

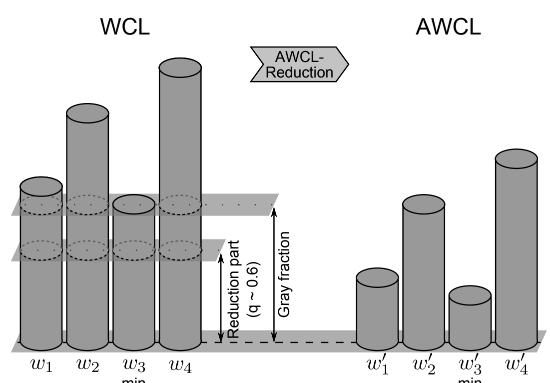

Figure 2. Illustration of the reduction, used by AWCL

Various reduction parts, referred to as q in equation (7), have been investigated, as described as follows.

$$P_b = \frac{\sum_{i\in\mathbb{V}} P_i * (w_i - q * min_{i\in\mathbb{V}}(w_i))}{\sum_{i\in\mathbb{V}} w_i - q * min_{i\in\mathbb{V}}(w_i)} \tag{7}$$

C. Weightage

Except from the plain CL algorithm, the presented coarse grained estimation techniques are utilizing weighting factors. The aim of weights is to give higher influence to more important (virtual) beacon nodes. In the given case a precalculation is defined as more important, if the accordant beacon node and therefore the linearizer is closer to the blind node. In the same way, it is more important if the number of beacon nodes included in the precalculation and in the blind node's communication range is high. Consequently the following weights have been studied:

Signal Strength – Virtual beacon nodes are weighted according to the RSS of the beacon node that provided the precalculation used to create the virtual beacon node. On average, the RSS is expected to be higher the closer the beacon node is. Although, variations of shadowing and fading may compromise this relation, it has been investigated as possible weightage. Equation (8) illustrates this weight, with i indicating the linearizer of the according precalculation as well as the resulting virtual beacon node.

$$w_i = RSS_i \tag{8}$$

Similarity – Virtual beacon nodes are weighted according to the rate of beacon nodes, included in precalculation, that are located within the communiction range of the blind node. This weight is given in equation (9), where $\mathbb{P}_i$ indicates the set of beacon nodes included in the precalculation of beacon node i and $\mathbb{B}$ indicates the set of beacon nodes within the communication range of the blind node. This is applied to the WCL approach, which is then called Similarity based WCL (SWCL).

$$w_i = \frac{|\mathbb{P}_i \cap \mathbb{B}|}{|\mathbb{P}_i|} \tag{9}$$

D. Reduction part

AWCL has been shown as more accurate than the original WCL. In advance of an included WCL estimation AWCL reduces all given weights by a certain portion of minimum weight, as given in equation (7). This leads to the behavior that in case of nearby weights the remaining small differences get more importance. For our investigations, the used reduction part q has been varied from 15% to 65%.

E. Distance approximation

To enable a blind node to use beacon nodes outside its own communication range, sDLS^{ne} introduced a distance approximation, given in Figure 3, that utilizes the given distance between linearizer and inaccessible beacon node (d_{iL}), and the estimated distance between blind node and linearizer (r_L), which was assumed to be as close as possible, due to the prior choice of the blind node. The sum of both distances is used as approximation of the unknown distance r_i.

Now, using not only the closest beacon node, but up to all beacon nodes within the communication range, this approximation tends to be more and more inaccurate. Therefore, two variants of this distance approximation have been investigated.

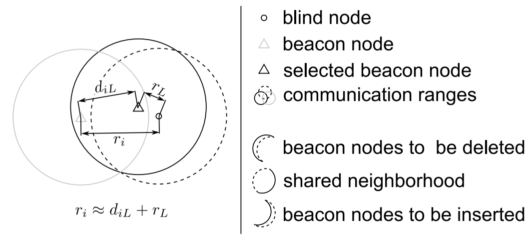

Figure 3. Approximation of a distance between blind node and inaccessible beacon node i by means of the linearizing beacon node L

Independent Approximation – For each precalculation distances to inaccessible beacon nodes are estimated as given in Figure 3. All data used is either directly estimated by use of measurements or provided by the precalculation itself.

Dependent Approximation – Most inaccessible beacon nodes are included in multiple precalculations, provided to the blind node. As illustrated in Figure 4, distance approximations towards such an inaccessible beacon node will differ according to the used precalculation, due to different linearizer nodes used in different precalculations.

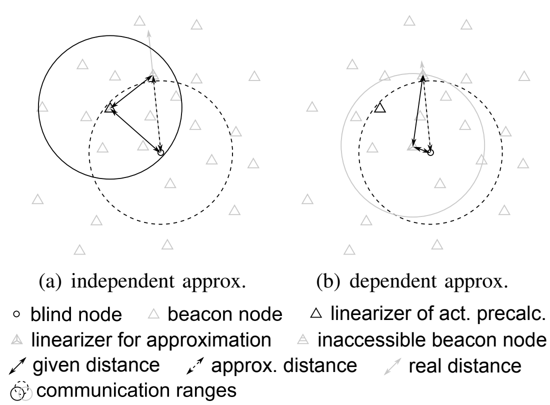

Figure 4. Two approximation strategies: To estimate distances of inaccessible beacon nodes, included in a precalculation, either (a) the beacon node, providing the precalculation or (b) the closest beacon node is used.

To provide the most precise distance estimation the shortest distance, which can be estimated from the given precalculations, have to be selected for calculation of virtual beacon nodes.

To achieve this, one possibility is to firstly determine all possible estimates for inaccessible beacon nodes, i.e., one for each precalculation, which includes the inaccessible node, to subsequently calculate the minimum distance estimations. In most cases a more efficient solution can be applied. Figure 5 illustrates such a solution compared along with independent approximation in the context of the over all position estimation, given on the right hand side. The illustrated approach

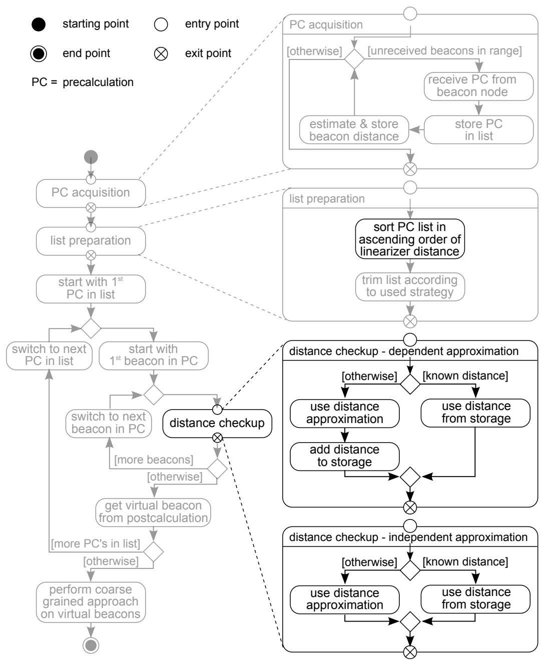

Figure 5. Differences of independent and dependent distance approximation (left), illustrated in the context of the HDLS algorithm (right).

processes precalculations individually but in ascending order of their distances towards the blind node, as illustrated on the right side. For this purpose, the distance between blind node and linearizer acts as the distance towards its according precalculation. Once a distance towards an inaccessible beacon node has been approximated by a close precalculation, this distance will be marked as known, as illustrated in the last but one box on the left side of Figure 5. If the same beacon node occurs in a further precalculation, the before calculated distance will be taken and the beacon node will be not treated as inaccessible.

Figure 4 illustrates the presented strategies by giving a worst case example for both approaches. Using independent approximation for the precalculation illustrated in Figure 4(a) highly overrates the distance towards the inaccessible beacon node. By use of dependent approximation instead, the better approximation provided by a closer precalculation illustrated in Figure 4(b) would be used.

IV. Simulations

To verify performance of the introduced HDLS approaches, the MATLAB® based network simulator Rmase is used [12]. The simulator provides a realistic radio communication model including spatial and temporal normal distributed fading. A static bidirectional spanning-tree routing was used to send data packets from nodes to sink and vice versa. Distance estimations performed by blind nodes rely on the simulators radio model.

A random deployment of n^2 nodes within a field of $n * n$ arbitrary distance units (adus) was utilized. The first node was always used as sink, while the remaining nodes have been randomly chosen as blind nodes (50%) or beacon nodes (50%). Note that the low number of blind nodes has been proofed to has no significant influence on the presented results but speeds up the simulation dramatically. The field size parameter n was varied from 5 to 30. The average communication range, given by the radio model, was 3 adus. For each field size the average over 100 simulations has been determined. In each simulated network all presented localization approaches have been performed concurrently.

V. Results

As described in Section III, there are various factors, influencing the accuracy of HDLS. To distinguish between the different approaches, resulting from these factors, a naming scheme is used, illustrated as syntax diagram in Figure 6. This diagram also shows the more than 350 combinations, which have been investigated by simulations. In Figure 6, "S" symbols similarity based weightage, applied to WCL. Reduction part of AWCL have been varied from 15% to 65%. Range based strategy, indicated with an "R", also denotes a percentage of the distance towards the closest beacon node, which limits the catchment area of further beacon nodes. It is used in addition to the fixed upper bound of virtual beacon nodes.

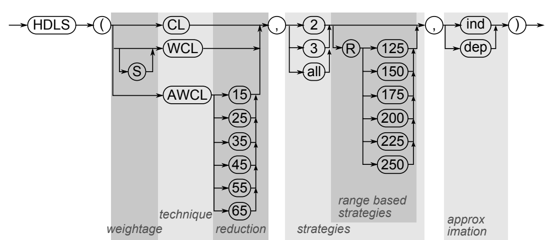

Figure 6. Syntax diagram: Naming of investigated HDLS approaches

First, used coarse grained techniques are analysed along with different virtual beacon strategies, i.e., the number of virtual beacon nodes. Figure 7 shows mean localization error over the number of deployed nodes, using the basic CL approach. It is illustrated, that the hybrid approaches perform significantly better than the underlying sDLS^{ne}, but in most cases not as accurate as the original sDLS approach with costly matrix updates. Furthermore, it is shown that the hybrid approach with two virtual beacon nodes is outperformed by the one, using three virtual beacon nodes. In contrast, using as much virtual beacon nodes as possible does not further increase localization accuracy.

Similar results have been found for HDLS based on WCL with traditional RSS based weighting, shown in Figure 8. It is shown that this approach performs the better, the more virtual beacon nodes are used. Futhermore, it outperformes

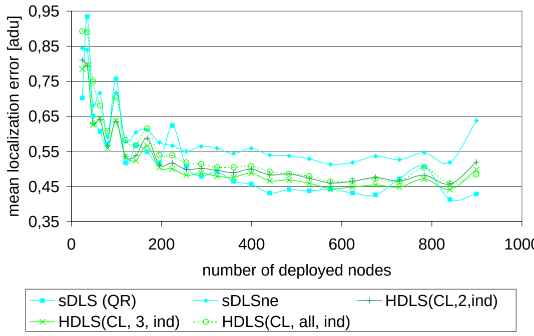

Figure 7. Mean localization error of CL based HDLS with independent distance estimation

sDLS and therefore it also outperforms the CL based approach.

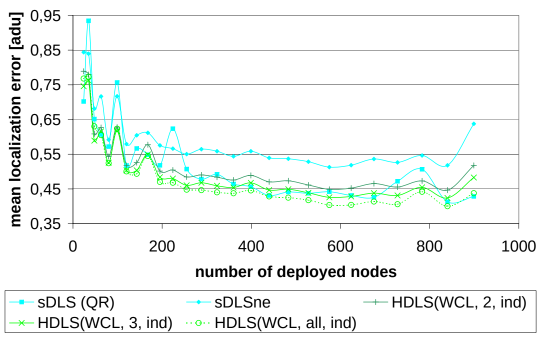

Figure 8. Mean localization error of WCL based HDLS with independent distance estimation

In Section III, SWCL has been introduced as an alternative approach of WCL, using similarity instead of signal strength. Both WCL based approaches are compared in Figure 9. On the one hand, the illustration shows that similar to the CL based approach, the SWCL approach performs best, when three virtual beacon nodes are used. On the other hand, it is shown that this approach is outperformed by the RSS based approach.

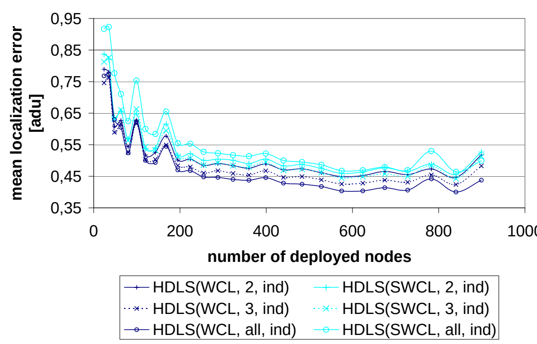

Figure 9. Mean localization error of HDLS based on WCL and SWCL with independent distance estimation

The third coarse grained technique, investigated to use with HDLS, is AWCL. The performance of AWCL depends on a reduction part, defined by AWCL. The best reduction part is said to be 55%. Therefore, this factor is also used for the results, given in Figure 10. The presented results show, that this approach also outperforms the costly sDLS approach and performs the better the more virtual beacon nodes are used. Further investigations, using different reduction factors showed that also in the given context a reduction factor of 55% performs best in most cases. Nevertheless, achieved accuracy is often influenced only marginal by the reduction factor.

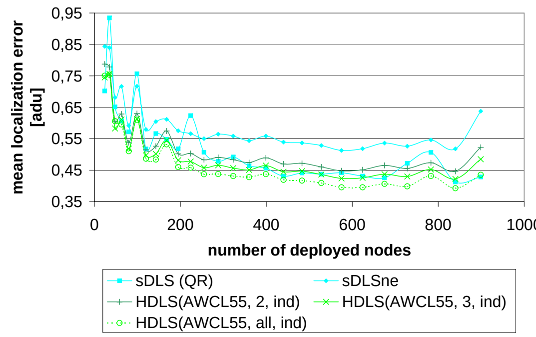

Figure 10. Mean localization error of AWCL based HDLS with independent distance estimation

As an intermediate result HDLS provides best accuracy, using as much virtual beacon nodes as possible, combined by AWCL with a reduction part of 55%. While the previous results used virtual beacon strategies with a fixed number of virtual beacon nodes, the following results investigate the range based virtual beacon strategy. The range within precalculations of beacon nodes are used to create virtual beacon nodes was varied from 125% to 250% of the distance between blind node and closest beacon node. The range based approach is combined with a fixed upper bound as presented before. Figure 11 shows the resulting localization accuracy for CL based HDLS, using various ranges and an upper bound of two, i.e., HDLS falls back into $\mathrm{sDLS}^{\mathrm{ne}}$, if the closest beacon node is significantly closer than all other beacon nodes. On the one hand, it is shown, that even a small range of 125% outperforms $\mathrm{sDLS}^{\mathrm{ne}}$. On the other hand, the graph shows, that only in few cases, this spatial limit outperforms the unlimited version. It also shows that in most cases a spatial limitation of 175% performs very close to the unlimited counterpart. Similar results have been found for the use of WCL, SWCL or AWCL.

As AWCL turned out as the most promising approach, it is selected to compare the range based strategy with various upper limits of virtual beacon nodes. Figure 12 shows the results for the previously introduced upper bounds in combiniation with the spatial limits of 125% and 250%. On the one hand, the results show that the range based strategy

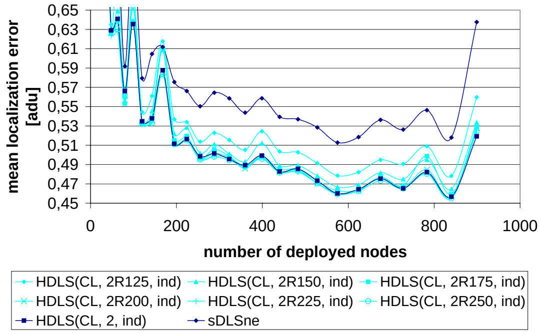

Figure 11. Mean localization error of CL based HDLS with independent distance estimation and range based virtual beacon strategy

also works for limits higher than two. On the other hand, it is shown that the higher the spatial limit, the lower the mean localization error. Although the unlimited approaches perform better than the corresponding limited approaches, it comes out that a spatial limit of 250% achieves good results.

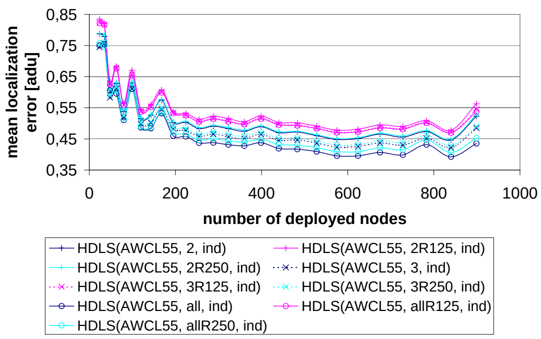

Figure 12. Mean localization error of AWCL based HDLS with independent distance estimation and range based virtual beacon strategy

To evaluate the range based strategy as an alternative to the before mentioned strategies of fixed limits, spatial limits have been figured out, which are equivalent to numerous limits. As illustrated in Figure 13, a spatial range of 150% can be put on a level with the upper bound of two virtual beacon nodes. A spatial limit of 200% instead can be equated with the strategy of using 3 virtual beacon nodes. Once again, as much beacon nodes as available is proved to provide lowest localization error. Nevertheless, a spatial limit of 250% provides also good results.

Using a spatial limitation instead of a fixed number of virtual beacon nodes can be only seen as alternative, if it is more cost efficient. Therefore, the number of arithmetic operations, used for the according localization approach, has been investigated. Figure 14 illustrates this cost for the HDLS approaches, presented in Figure 13. It clearly comes out that the two range based approaches, which have been pointed out as equivalents need slightly more computations than the corresponding approaches.

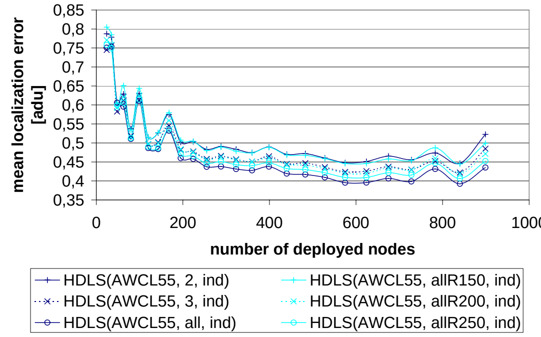

Figure 13. Range based equivalents of AWCL based HDLS approaches with independent distance estimation – mean localization error

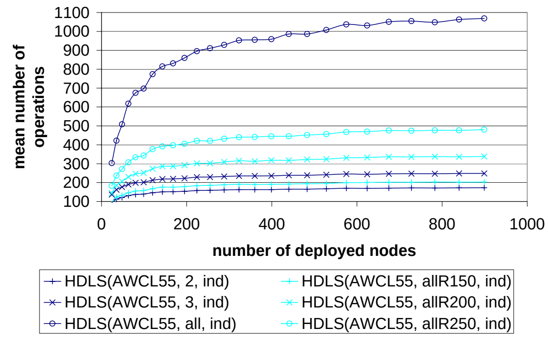

Figure 14. Range based equivalents of AWCL based HDLS approaches with independent distance estimation – mean number of operations

Up to this point, the presented results are based on independent approximation of distances towards inaccessible beacon nodes. The remaining part of this section presents the results, achieved by use of dependet approximation. As shown in Figure 15, use of this approximation significantly improves localization accuracy of CL based HDLS. It outperforms sDLS as well as the best CL approach with independent approximation, even if only two virtual beacon nodes are used. It is also shown that there is only a small gain, which distinguishes the all beacon strategy from the three beacon strategy. Similar results have been found using WCL, SWCL and AWCL. In all cases, each approach using dependent distance approximation outperforms the according HDLS approach based on independent distance approximation, using as much virtual beacon nodes as possible.

To sum up the before mentioned results and to figure out the best HDLS approach for each coarse grained technique the best performing approach is presented in Figure 16. Noticeable, but not surprising, best results are achieved using as much virtual beacons as possible. Furthermore, Figure 16 shows impressively that use of dependent distance approximation outperforms independent distance approximation. Using dependent approximation, the AWCL based approach performs best closely followed by WCL. The same

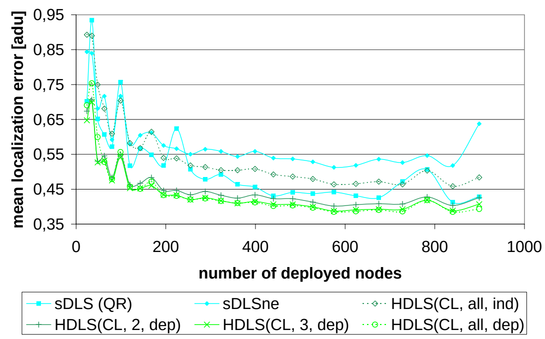

Figure 15. Mean localization error of CL based HDLS with dependent distance estimation

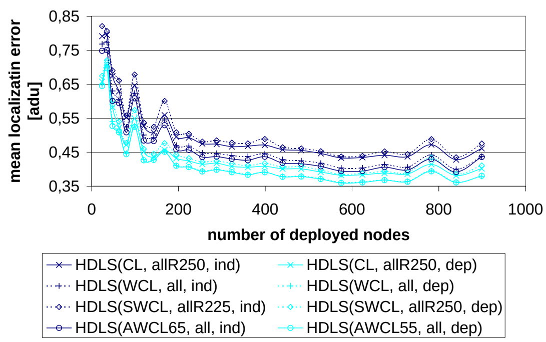

Figure 16. Mean localization error of best performing HDLS approaches, grouped by estimation technique and distance approximation

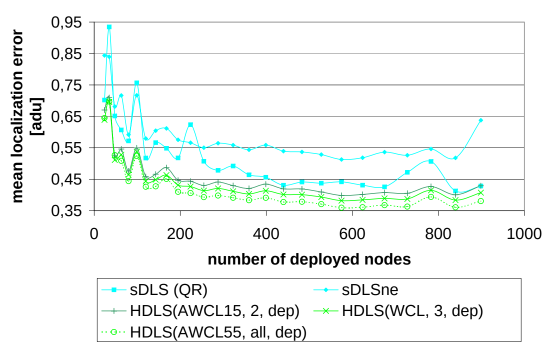

Figure 17. Mean localization error of best performing HDLS approaches for different limits of virtual beacon nodes

order is depicted for independent distance approximation. Furthermore, it is shown, that range based virtual beacon strategy is useful in combination with CL and SWCL. In all cases a high spatial limit is used.

Due to the obviously strong impact of the number of virtual beacon nodes, same analyses have been performed, taking only results with an upper limit of three or two virtual beacon nodes into account, respectively. In both cases dependent distance approximation outperforms independent distance approximation. Also the internal order of the presented approaches is similar to the one presented in Figure 16. For each upper limit of virtual beacon nodes the best HDLS approach is presented in Figure 17. As it is illustrated, two times AWCL based approaches provide best results, while in the third case WCL performs best. All given HDLS approaches perform better than original sDLS with costly update operations. Even though, using as much virtual beacon nodes as possible results in highest accuracy, high accuracy can be also achieved using two or three virtual beacon nodes.

The achieved improvements in localization accuracy are mainly caused by an increased number of beacon nodes, used for localization. Due to the fact, that different virtual beacons, based on different precalculations, use different sets of beacon nodes, cardinality of resulting unions is commonly higher than cardinality of the individual sets. Since the number of used beacon nodes only depends on the applied virtual beacon strategy, Figure 18 exemplary shows the number of used beacon nodes for the best cases, presented in Figure 17. It is shown, that using two virtual beacon nodes increases the number of beacon nodes used by about 40% compared to sDLS^{ne}. Using three virtual beacon nodes leads to an increase of about 67%, while using as much virtual beacon nodes as possible leads to an increase of 245% beacon nodes.

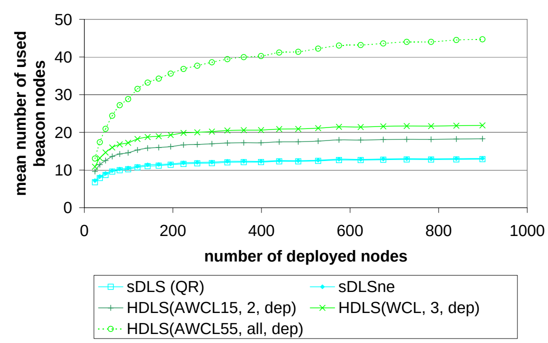

Figure 18. Mean number of used beacon nodes of best performing HDLS approaches for different limits of virtual beacon nodes

As a matter of course, using more beacon nodes, increasing localization accuracy, comes along with increased cost by means of computation. The mean number of operations, performed on each blind node, to perform one localization is given in Figure 19. The number of operations is mainly determined by the number of virtual beacon nodes or the number of individual precalculations, respectively. The most important result is that all presented approaches need less computations than the original sDLS with matrix updates, while all of these approaches, given in Figure 19, provide higher accuracy than sDLS. The additional cost for each additional virtual beacon node is about 80% of the cost of sDLS^{ne}.

VI. Conclusion

In this work, the efficient localization approach sDLS^{ne} has been combined with various coarse grained approache

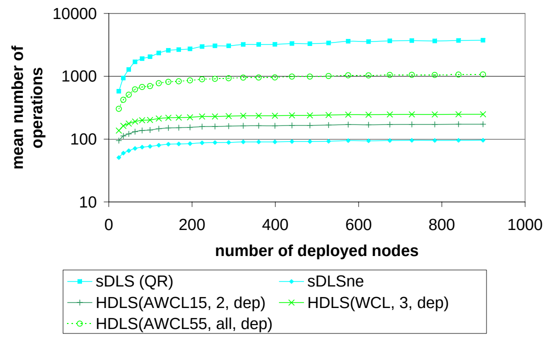

Figure 19. Mean number of operations of best performing HDLS approaches for different limits of virtual beacon nodes

to improve accuracy of localization. As shown in Figures 8, 10, 15, and 17, the new HDLS approach provides higher accuracy than sDLS^{ne} and even outperforms the initial sDLS approach. Using the newly introduced dependent distance approximation, even use of only two virtual beacon nodes, i.e., two precalculations, dramatically increases localization accuracy. Although HDLS needs more computations than sDLS^{ne}, it needs much less computations than sDLS. It further provides the possibility to chose between various variants with different cost. Using a small range, the presented range based virtual beacon strategy provides an very cost efficient way to improve sDLS^{ne}.

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